Unemployment insurance management: A real option approach.*

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Abstract

In absence of reinsurance market for social insurance risk, managed by government, we propose to resort to existing financial markets in order to hedge the unemployment insurance risk. We identify a portfolio of traded assets as a relevant virtual underlying security to this risk. This construction allows us to apply contingent claim analysis to such a non-traded asset as social Insurance.

Key words : Real options, risks, functional correlation coefficient, unemployment insurance risk.

JEL classification n°: C13, C14, D81, G12, G13, G22.

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Introduction

Risks diversification in an insurance company portfolio is not perfect, consequently companies face a risk. To reduce this risk, they exchange contracts or part of them on a reinsurance market. The main model of reinsurance market equilibrium is due to K. Borch [2]. This risk transfer applies easily to direct private insurance companies but is less obvious concerning social insurances. Indeed, reinsurance markets can exist for private social insurance companies but does not exist for public social protection in so far as the government is commonly considered as its own insurer.

The social protection systems, established by most of the European country governments, insure large risks, non-diversifiable, that are not be assumed by private companies usually. Moreover, the solidarity role of public social insurance is an important feature: agents facing different risks are insured under the same conditions. Avoiding adverse selection risk leads to the *obligatory* contributions. One can refer to Ewald [6] for a detailed analysis of the social insurance history. These social insurance systems are managed by public organisms and the possible global deficits are often made up by the government (government as *payer of last resort*). In this paper, we are interested in the French unemployment insurance system, managed by the social partners, with equal representation of both trade union and employers syndicate[1]. The only financing source is the workers and employers monthly obligatory contributions. In the following, we consider without loss of generality, that the government directly manages the unemployment insurance. Actually, the budgetary policy is the main tool of the risk unemployment insurance management. But, this approach has many limitations and does not solve problem of risks diversification.

As an alternative, Kast and Lapied [7] propose to resort to international diversification by creating a market of claims contingent to the unemployment rates in different countries. The principle of these so-called *u-bonds* is based on the decentralized transfer of returns from economies in high part of business cycle to economies in the low part. These contingent claims are bonds with payments indexed on unemployment rates. In order to make the management of such contingent claims easier, the authors propose to organize a complete set of derivative assets markets on the *u-bonds*. This approach refers to the securitization of risks and can be compared to the creation of the *cat-bonds* and the *weather-derivatives* markets. Typically, such a market would play the role of reinsuance markets.

In absence of this kind of reinsurance markets, natural alternative seems to be the existing financial markets. Concretely, governments do not use financial markets to reinsure but ponctually to issue fixed rate debt. For example, between 1989 and 1993, the French unemployment insurance system was in deficit coming up to FRF[2] 35 billions at the end of 1993. French government contributed to the accounts recovery by issuing bonds. Obviously, these bonds do not provide a perfect hedge to the unemployment insurance risk.

In this work, we propose to integrate the resort to financial markets in a global management of unemployment insurance. Indeed, as the unemployment insurance risk can be viewed as a risky asset for the government, the idea is to value and/or to hedge this non-traded asset using traded securities. This approach is suggested by recent developments in real investment theory called 'real option theory '(Dixit and Pendyck[5]). Real option theory is an answer to the limitations of traditional investments valuation methods such as Net Present Value. This theory allows to take uncertainty, irreversibility and possible flexibility of decisions into account. It is based

[1] It was still the case during the observation period.

[2] French Franc was the currency in circulation during the observation period. FRF 1 is worth EUR 0.15245

on the analogy between possibility to postpone investments (investment opportunities) and a financial call option. Questions of flexibility of decisions are not our purpose here. However this theory allows us to value and to hedge an investment such as contingent claims on an underlying asset by using extensions of the Black-Scholes formula [1]. Applying this method to the social insurance risk would allow public or private insurance companies to hedge this risk on financial market and/or to value this asset as if there were a reinsurance market.

Real-options theory has been applied to value copper mines, oil sinking, etc., for which there exists a natural relation between a marketed asset and a risky asset to be valued (*future* on copper, on oil[3]). In this case, contingent claim valuation *à la Black-Scholes* directly applies. Unfortunately, in many cases, there is no obvious asset connected to the risky asset to be valued. Typically, this situation is the case of unemployment insurance risk. Indeed, it can be viewed as a derivative asset of the unemployment rate which is not a traded asset. Moreover, there is no obvious traded asset underlying the unemployment rate unless a company stock would be perfectly correlated with this unemployment rate, which seems unrealistic. Kast, Lapied and Pardo [8] propose a methodology to construct a portfolio of traded assets which plays the role of a virtual security underlying a non-traded risky asset. In this paper, we apply this method to the unemployment insurance risk. We recall the principle and the method in the two first sections. The third section is an application to real French unemployment allocations data.

1 Identification of a portfolio as a virtual underlying security

The main requirement of contingent claim analysis is that uncertainty over the future values of a risky asset can be replicated by marketed assets. The spanning assumption holds, for instance, for most commodities which are traded on both spot and futures market. Nevertheless, many situations do not satisfy this hypothesis (See for instance, Cortazar, Schwartz and Salinas[4]). It is indeed the case for the unemployment insurance risk. To overcome this difficulty, we propose to identify a portfolio of available assets as a virtual underlying asset. As a consequence we want to construct a portfolio perfectly correlated with the unemployment insurance risk. A deterministic function relating an underlying asset and an uncertain payoff process is sufficient to apply derivative asset pricing theory in the spirit of Black and Scholes [1] and Merton[10]. The problem is the identification of a pertinent underlying asset. We briefly recall a result allowing the uncertainty identification of a non-traded asset as a traded underlying one.

Let $(\Omega, \mathcal{F}, P)$, be a filtered probability space. Let $W^1, ..., W^n$ be n independant standard Brownian motions on $(\Omega, \mathcal{F}, P)$. The filtration $(\mathcal{F}(t))_{0 \leq t \leq T}$ is defined as the natural filtration generated by $W^1, ..., W^n$. Let $Z(t)$ and $\tilde{Z}(t)$ $0 \leq t \leq T$, be two Brownian motions on $(\Omega, \mathcal{F}, P)$ such as $\mathcal{E}(dZd\tilde{Z}) = \rho dt$ (ρ is a constant correlation coefficient). Let X be the payoff process of the non-marketed asset to be valued and let Y be the payoff process of a marketed asset.
If we assume the existence of a deterministic function between X and Y at each date, then X can be considered as a derivative on the underlying Y. Finding such a Y and a deterministic function is a means to identify the source of uncertainty for X as a marketed one. From there onwards real options theory can be applied to X. We consider X and Y price processes, with

[3]See for instance Brennan, Schwartz[3], Slade[12]

dynamics given by:

$$dX_t = \mu_X X_t dt + \sigma_X X_t d\tilde{Z}_t$$

$$dY_t = \mu_Y Y_t dt + \sigma_Y Y_t dZ_t$$

and $Y(0) = y_0$[4]. μ_X and μ_Y are instantaneous trends, and we assume that standard deviations σ_X and σ_Y are strictly positive. Our problem is to identify the Brownian motion generating X process. Alternatively, we look for Y in such a way that there exists a deterministic function φ such as for all t:

$$ln\left(\frac{X_{t+\Delta}}{X_t}\right) = \varphi\left(ln\left(\frac{Y_{t+\Delta}}{Y_t}\right)\right) \tag{1.1}$$

The following theorem[5] characterizes a deterministic functional relation between prices (here, between log-prices) using the correlation coefficient.

Theorem 1.1
Let $\frac{dX}{X} = \mu_X dt + \sigma_X d\tilde{Z}$, and let $\frac{dY}{Y} = \mu_Y dt + \sigma_Y dZ$, with constant correlation coefficient ρ. If $\exists t\ 0 \leq t \leq T$, $\exists \Delta$ a time increment and $\exists f\ \Re \to \Re$ non-degenerated such as:

$$\frac{X_{t+\Delta}}{X_t} = f\left(\frac{Y_{t+\Delta}}{Y_t}\right)$$

then $|\rho| = 1$

Identifying Y as the source of uncertainty for X amounts to say that the two processes are perfectly correlated.

2 Methodology.

In case where there is no obvious marketed asset Y connected with X, we will construct Y as a portfolio of marketed assets, such as $ln\left(\frac{X_{t+\Delta}}{X_t}\right) = \varphi\left(ln\left(\frac{Y_{t+\Delta}}{Y_t}\right)\right)$, with φ is a deterministic function. In this section we briefly present the construction methodology proposed in Kast, Lapied and Pardo [8]. This construction is based on the maximization of the functional correlation coefficient (FCC) between the non-traded asset and the optimal portfolio. Indeed, let X and Y be two random variables, the existence of any functional relation is given by a ratio of correlation of Y in X equal to 1. The FCC is defined by:

$$\eta^2_{Y|X} = \frac{\mathbf{V}\left(\mathbf{E}\left(Y|X\right)\right)}{\mathbf{V}\left(Y\right)} \tag{2.1}$$

where $\mathbf{V}$ is for variance.
FCC takes values in $[0,1]$, and $\eta^2_{Y|X} = 1$ is equivalent to the existence of a deterministic function f so that $Y = f(X)$ almost surely (Saporta [11]). In order to construct an empirical version of $\eta^2_{Y|X}$, in the case where X and Y are continuous, one only has to estimate, in a first step, conditional expectations i.e. regressions $g(X) \overset{def}{=} \mathbf{E}(Y|X)$. The construction used is the non-parametric method of the orthogonal functions.

[4] Concerning X process, we should have whether initial or final conditions, according to the problem to be solved.

[5] Proof of this theorem is given in Kast, Lapied, pardo[8]

To answer our problem of construction, we consider prices of N financial traded assets $\{S^1, ..., S^N\}$ with payoffs evolving according geometric Brownian processes:

$$dS_t^i = \mu_i S_t^i dt + \sigma_i S_t^i dZ_t^i$$

where μ_i are instantaneous trends, σ_i are standard deviations and dZ^i are increments to Gauss-Wiener processes such as $\mathcal{E}(dZ^i dZ^j) = \rho_{ij} dt$. With these assumptions the prices S_t^i are lognormally distributed.
Note $R_{it} = ln\left(\frac{S_t^i}{S_{t-1}^i}\right)$ the continuously compounded return of asset i at time t.
On the other hand we consider the non-traded asset X as defined in the previous part:

$$dX = \mu_X X_t dt + \sigma_X X_t d\tilde{Z}_t$$

$d\tilde{Z}$ is increment to Gauss-Wiener processes such as $\mathcal{E}(dZ^i d\tilde{Z}) = \rho_i dt$. The non-traded asset X is correlated, but not perfectly, with the available traded assets ($\rho_i \neq 1\ \forall i$). Note $I = \{1, ..., N\}$ and let R_{Xt} be the continuously compounded return of the non marketed asset X at time t.
In order to identify the uncertainty source of X as a portfolio of traded assets, we construct:

$$R_{Yt} = \sum_{i=1}^{N} \lambda_i^* R_{it} \tag{2.2}$$

such that:

$$(\lambda_i^*)_{i\in I} = \max_{(\lambda_i)_{i\in I}} (\eta^2_{R_Y|R_X}) \tag{2.3}$$

with $\sum_{i=1}^N \lambda_i^* = 1$

The portfolio satisfies:

Proposition 2.1

$$Y_t = \frac{Y_0}{\prod_{i=1}^N (X_0^i)^{\lambda_i}} \prod_{i=1}^N (X_t^i)^{\lambda_i}$$

is solution of: $R_{Yt} = \sum_{i=1}^N \lambda_i R_{it}$

Prices lognormality allows to conclude that dynamic of the portfolio Y evolves according to a geometric brownian motion $dY_t = \mu_Y Y_t dt + \sigma_Y Y_t dZ_t$ with parameters calculated by:

Proposition 2.2
According the previous assumptions, the portfolio Y follows a geometric brownian motion:

$$dY = \left[\sum_{i=1}^N \mu_i\lambda_i + \frac{1}{2}\sum_{i=1}^N \sigma_i^2\lambda_i(\lambda_i - 1) + \frac{1}{2}\sum_{i\neq j}\rho_{ij}\sigma_i\sigma_j\lambda_i\lambda_j\right] Y dt + \sqrt{(\sum_i\sum_j \sigma_i\sigma_j\lambda_i\lambda_j\rho_{ij})} Y dZ$$

where dZ is a Gauss-Wiener

According theorem 1.1, we know that this portfolio returns are perfectly correlated with those of X. With this optimal portfolio, the deterministic function φ satisfying:

$$R_{Xt} = \varphi(R_{Yt})$$

can be estimated using the same non-parametric method[6]. For further details, see Kast, Lapied, Pardo[8].
In conclusion, this method allows to identify a portfolio of traded assets as a relevant virtual underlying security to the non-traded asset X. This portfolio can be used to hedge the risk X or to have an idea of the price that would have X if it were traded on a financial market.
The question we are interested in here, is the hedging of the unemployment insurance risk. The previous methodology applied to this risk will allow us to identify a portfolio of traded asset as a virtual underlying security. This portfolio can be used to build a hedging strategy and to estimate the price which the unemployment insurance risk would have on a reinsurance market to be.

3 The unemployment insurance risk management

In this section we empirically identify a portfolio as a relevant virtual underlying asset to the French unemployment insurance risk. We use macroeconomics data of French unemployment allocations monthly paid by the social partners. We have 117 monthly observations, from 01/01/1989 to 09/01/1998. In order to construct a portfolio, we need financial data corresponding to the same period. Data are constituted by monthly returns of 30 financial assets of CAC 40, observed the first of each month, for which complete series are available from 01/01/1989 to 09/01/1998. We work with the logarithm of these returns. (Reliability of this empirical method is established on financial data in Kast, Lapied, Pardo, Protopopescu [9]).
We note R_{it} the geometric return of traded asset i at time t, and R_{Y^*}, the geometric return of the optimal portfolio. We construct this optimal portfolio as a relevant virtual underlying asset for the unemployment allocation returns with the 30 assets extracted from CAC 40.

3.1 Portfolio Construction

We construct the optimal portfolio that maximizes the FCC between this portfolio and the allocation returns, i.e we construct Y^* such that $R_{unemp} = \varphi(R_{Y^*})$ where R_{unemp} are the unemployment allocation logarithmic returns. The optimal portfolio calculated, we estimate the functional relation $\tilde{\varphi}$ such as:

$$R_{unemp} = \tilde{\varphi}(R_{Y^*}) \tag{3.1}$$

This construction yields an estimation of the unemployment allocation process $\tilde{X}_t$:

$$d\tilde{X}_t = \mu_{\tilde{X}}\tilde{X}_t dt + \sigma_{\tilde{X}}\tilde{X}_t dZ_{Y^*} \tag{3.2}$$

Another method would be to estimate the asset returns using the CAPM approach. But, the CAPM approach does not say anything about the functional relation between the portfolio and the non-traded asset. Next tables compares the quadratic error between estimated logarithmic returns of X obtained with the FCC method and the CAPM method to X observed returns.

EQ_η	EQ_{CAPM}
0.0015	0.0046

[6] In fact, the estimation is done in two steps.

The error using the FCC is given by :

$$EQ_{\eta}(i) = \frac{1}{T}\sum_{t=1}^{T}(R_{it} - \tilde{\varphi}_i(R_{Y_t^*}))^2 \tag{3.3}$$

where $\tilde{\varphi}_i$ is the estimate regression function of R_i on R_{Y^*}.
The quadratic error for the CAPM is :

$$EQ_{CAPM}(i) = \frac{1}{T}\sum_{t=1}^{T}[R_{it} - \beta_i R_{CAC,t} - r_0(1-\beta_i)]^2 \tag{3.4}$$

where:

- β_i is the "bêta" of asset i, i.e sensitivity of asset risk i to the CAC40 one;
- $(R_{CAC40,t})_{t=\overline{1,T}}$ is the CAC40 rates of return vector;
- $(1+r_0)^T = \prod_{t=1}^{T}(1+r_t)$, where r_t is the monetary rate at time t ;

The next two figures compare the unemployment allocation returns (dotted line), on the one hand to those of the portfolio constructed by FCC method, (solid line) (Figure 1) and on the other hand, to those calculated by the CAPM method (R_{CAPM})(solid line) (Figure 2).

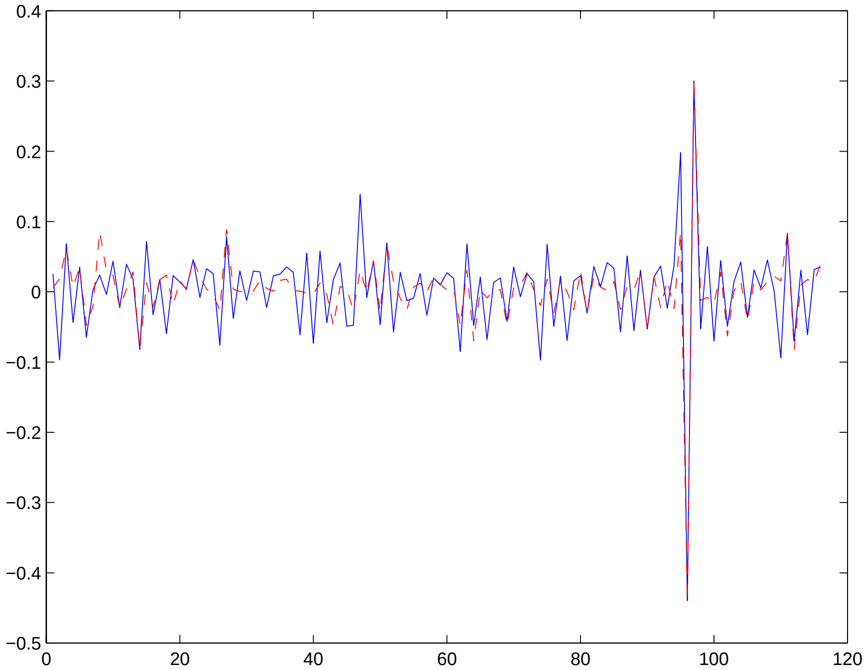

Figure 1: Comparison between $\varphi(R_{Y^*})$ and R_{unemp} for FCC method

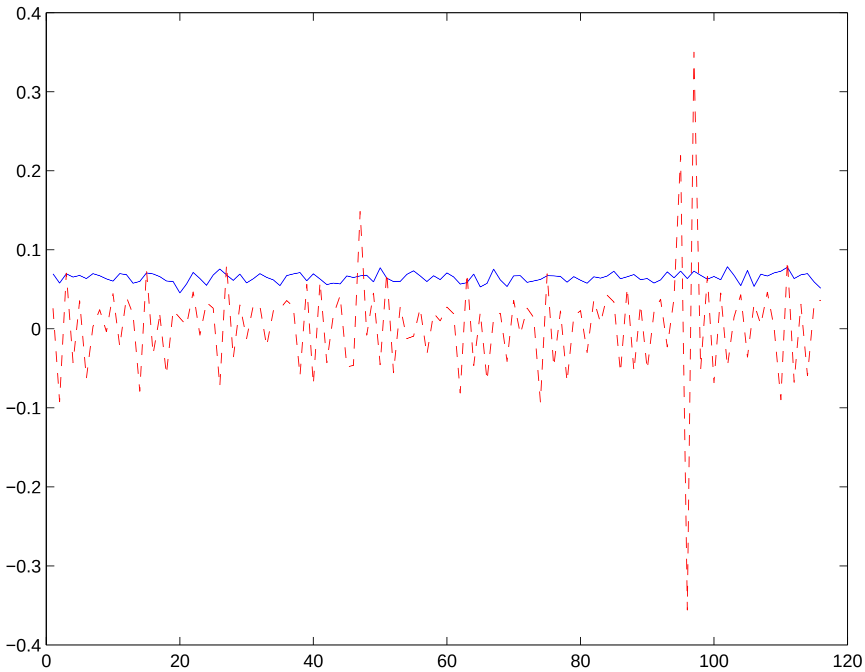

Figure 2: Comparison between R_{CAPM} and R_{unemp} for CAPM method

Comparison between these two figures shows that the portfolio returns obtained by the FCC method is a better approximation than those calculated with the CAPM. It is confirmed by the mean quadratic error that is lower for the FCC than for the CAPM. This results seem to be obvious because the CAPM is based on the covariance between the CAC 40 and the unemployment allocation and there is no reason that all the risk of the unemployment allocation be explained by the CAC 40[7].
The portfolio constructed as a virtual underlying asset to the unemployment insurance risk can be used to construct a hedging strategy using contingent claim analysis.

3.2 Risk premium

In this part, we want to calculate the unemployment insurance risk premium, i.e the risk premium of the asset $\tilde{X}$ if it were marketed on a financial market. It can be seen as what risk premium should the government used if it issued a new bond to hedge unemployment insurance risk.
$\tilde{X}$ process satisfies the diffusion equation:

$$d\tilde{X}_t = \mu_{\tilde{X}} \tilde{X}_t dt + \sigma_{\tilde{X}} \tilde{X}_t dZ_{Y^*}$$

The standard estimation method of the drift and volatility is the maximum likelihood method (ML), using the relation between these parameters and those of the logprices satisfying:

$$d(\ln \tilde{X}_t) = (\mu_{\tilde{X}} - \frac{\sigma^2_{\tilde{X}}}{2})dt + \sigma_{\tilde{X}} dZ_Y \tag{3.5}$$

The maximum likelihood method provides the estimator of the mean and the variance of $R_{\tilde{X}_t} = \Delta log\tilde{X}_t$:

$$\tilde{m} = \tfrac{1}{117} \textstyle\sum_{t=1}^{117} R_{\tilde{X}_t}$$

$$\tilde{v}^2 = \tfrac{1}{117} \textstyle\sum_{t=1}^{117} (R_{\tilde{X}_t} - \tilde{m})^2$$

The ML estimators of the drift and volatility parameters are deduced from their relations to the mean and variance parameters. They are given by:

$$\hat{\mu}_{\tilde{X}} = \tilde{m} + \tfrac{\tilde{v}^2}{2}$$

$$\hat{\sigma}^2_{\tilde{X}} = \tilde{v}^2$$

Let us note $\tilde{a}$ the risk premium of the unemployment risk, it is given by the difference between the yearly rate of return of $\tilde{X}_t$ and the continuously compounded risk-free rate.
Next table gives the results for three different values of r

	$r = 4\%$	$r = 5\%$	$r = 7.19\%$
$\tilde{a}$	1.23%	0.23%	−1.89%

Risk premium

The risk premium depends on the risk-free rate. As a result, we see that the risk premium of the unemployment risk is small: this is consistent with the slight volatility of the unemployment allocation process (4% par annum). The $\tilde{a}$ negative case, or equivalently $\mu_{\tilde{X}} < r$, is theoretically irrelevant because we are supposed to compare a risky asset to a risk-free asset. It could be

[7] CAC 40 is the main index on French market

explained by the fact that, in practice, r is the rate of return of an asset which is not risk-free, hence has a non-zero volatility, and can even have a volatility such that $\sigma_r > \sigma_{\tilde{X}}$. For instance, $r = 7.19\%$, e.g the mean yearly monetary rate, continuously compounded with the rates on this period, visibly corresponds to a risky asset rate of return. Moreover $\tilde{X}$ is not traded on a financial market, therefore $\mu_{\tilde{X}} < r$ does not correspond to an arbitrage opportunity. In this case, we conclude that $\tilde{X}$ is not marketable, and we cannot value the risk, but this does not imply that its price is zero. On the other hand, in this example μ_{Y^*}, the rate of return of the portfolio we have constructed is higher than r, therefore we can use it as an underlying security in classical applications. But, if we had μ_{Y^*} smaller than the risk-free rate, meaning that σ_r[8] $> \sigma_{Y^*}$, we would reach the limits of the model.
We have calculated the risk premium by the CAPM formula, and found it close to zero, but negative, for each value of the risk-free rate. We can conclude that, from the CAPM point of view, the unemployment risk is negligible and does not need a financial management. We have seen with the previous method that this result has to be modulated in regard to the risk-free rate's values. This confirms the limits of the CAPM method concerning non traded risks.

We have constructed a process identifying the unemployment risk and a market risk. If the risk premium is higher than zero, the unemployment risk could be managed by financial instruments. On the other hand, if it is negative, it is not possible to hedge this risk on financial markets. In any case, the process constructed can be used in optimal control problems as, for instance, the optimal time to switch to another compensation system.

Concluding Remark

In this work we identify a portfolio of traded assets as a relevant virtual underlying security to the unemployment insurance risk. This construction allows us to apply contingent claim analysis to such a non-traded risky asset as social insurance. This portfolio can be traded by government and can be a way to hedge the unemployment insurance risk.

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[8] σ_r corresponds here to the global risk of the country that faces up to the unemployment risk

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