Bargaining and the Penalty for Conversion
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Abstract

A penalty for conversion in the insurance of real property exists when the indemnity received by a client is reduced because she decides not to restore the structure as it existed before damage occurred but instead to build a different structure on the site. The problem has been viewed as one of optimum ex ante contracting by Bourgeon and Picard. In this paper it is examined as a product of ex post bargaining. The paper shows that bargaining undermines some functions of insurance and strengthens others. Insurers with strong bargaining power are sometimes preferable to consumers ex ante because they tend to limit a problem of over insurance. Results are derived under the generalized Nash bargaining solution and under a constant shares solution.

1 The penalty for conversion

Consider situations like the following one: When fire heavily damages a building, the insurer stands ready to restore the property to its previous use and condition. The client wants instead to convert the structure to another use. The insurer agrees to the conversion with the additional provision that the indemnity paid to the client is substantially less than the cost of restoration. Thus the insurer tries to exact a penalty for conversion. The client objects to the penalty, demands a greater indemnity, and threatens legal action.

Threats, negotiations, and penalties for conversion stand outside the classic model of insurance, but they are possibilities whenever a client wants to convert a damaged property to something new. For the benefit of the theory, and to improve insurance practice, some account should be given of the penalty for conversion and of the other aspects of these situations. The basic question is: How should the penalty for conversion be understood?

Jean-Marc Bourgeon and Pierre Picard (2001) understand the penalty for conversion as the solution to a problem that is inherent in the insurance contract. The problem is that the value of converting the structure is private information known only to the client and not known to the insurer or the courts. Because it is private, the information cannot be used in the contract. It is convenient here to express their idea in language borrowed from the health insurance literature: the value of conversion is not contractible meaning that it is not enforceable at reasonable cost by a civil court.

Motivated by these considerations, Bourgeon and Picard describe a second-best contract. Instead of depending on the value of conversion, the contract depends on a signal. The client signals the privately known conversion value by either converting the structure or restoring it. When she converts the structure, she signals that the value of conversion is high. In such cases, on average, she is relatively rich, and being risk-averse, she seeks a lesser indemnity. When the client restores the structure she signals that conversion is not very valuable and that she is relatively poor. In her optimum contract the indemnity is higher. The penalty for conversion is the difference between the indemnities. Since it addresses a problem of excessive indemnities, the penalty might as accurately be called a discount. The signal is contractible, but because it is a binary signal of a continuous variable, it is imperfect. Thus in the analysis of the second-best contract, the penalty or discount for conversion is a device by which the insurance contract combats excessive indemnities.

More work is needed on penalties for conversion. The settlement of insurance claims is in practice an intense negotiation. The terms of the contract are not fully decisive, regardless of whether they are ex ante optimum. In many instances the settlement process involves a protracted period of negotiation, bargaining, and legal maneuvering. Take for instance the case of the suspicious fire in the Paris building of the Credit Lyonnais, a case cited by Bourgeon and Picard. The affair features negotiation, delay, and threat, to such an extent that bargaining is as prominent a feature as contracting. As another instance, after the 1994 Northridge earthquake numerous insurers were charged under the California Unfair Claims Practices Act with negotiating settlements in bad faith. The insurance commissioner was forced to resign because of his failure to punish and redress the abuses. Bargaining is a feature of property insurance that a comprehensive theory should take into account. It is, moreover, a source of penalties for conversion, as explained below.

A second reason for further work on penalties for conversion is the doubt concerning whether damage is always a contractible quantity. The answer is not obvious. Damage is, by definition, the cost to restore the property. The idea that it is contractible is reasonable when the amount of damage can be confirmed by restoring the property and presenting the receipts to the court. When the client chooses to convert the property, her choice makes the amount of damage debatable. Damage is no longer a contractible quantity here because either party can bring to court contractors, estimators, and other experts to give testimony to a wide variety of values, as is seen routinely in condemnation proceedings. The indeterminacy of the outcome and its expense mean that damage is non-contractible when the structure is converted. Thus theoretical concerns are also important in motivating further research on penalties for conversion.

2 Setting

Consider the typical character of urban real property. A new structure is usually well suited to its site but the typical structure is to some large or small degree worn, obsolescent, not quite up to code, and inappropriate to its site. It is not what would be built if the land were vacant today.

When the structure suffers damage the wealth of the owner falls, but there is a limit to how far it falls. Beyond a certain level of damage the best policy is to clear the site and build the current best structure. The amount of damage that just triggers conversion is called the conversion threshold and is denoted by q. Because the net value of conversion is the land value, the conversion threshold is the difference between the value, v, of the land and structures, and the value of vacant land, l. Thus

$$q = v - l$$

For instance, a home that sells for $v = \$300,000$ while occupying land that would if vacant sell for $l = \$100,000$ has a conversion threshold of $q = \$200,000$. Damage less than $\$200,000$ provokes repairs. Damage beyond the threshold provokes conversion. Damage can exceed the threshold. The cost of restoring the structure might be $t =$

$\$250,000$, which is more than value of the incumbent structure, or even $t = \$400,000$, which is more than the value of the restored property. Such outcomes are typical when the incumbent structure is not a new building perfectly suited to its site.

Now consider insurance of the property. The contract pays an indemnity equal to damage, which is regarded provisionally as always contractible. Then the consumer requires the contact to have an upper limit at $\$200,000$. The upper limit is absolutely needed because damage can surpass $\$200,000$, in which case, in the absence of an upper limit, the client becomes over insured. The risk-averse client avoids such gambles. The upper limit chosen by the client is denoted by b, and the nominal indemnity paid when damage is t is $\min[t, b]$. Because damage is contractible only up to the threshold, this nominal indemnity turns out to be no more than a starting point for bargaining.

Real world contracts are written in terms of damage and they have upper limits. This section and the next discuss the reasons why, but contracts in damage with upper limits are the rule in any event. In modelling the contracts, the deductible can be safely ignored. Actual clients choose deductibles that are small in comparison to the value insured, and the deductibles do not vary much with changes in wealth and tastes. Because pricing is fair, coinsurance appears only to the extent that it is implicit in the bargaining solution. Such models of property insurance are considered in a series of papers. A theoretical examination is found in Garratt and Marshall (2001B). Applications appear in Garratt and Marshall (1996, 2001A, and 2001C).

3 Knowledge, Contracts, and Enforcement

An insurance contract on real property is written in terms of damage t. That is an empirical fact, and it is comprehensible in the theory. To understand why damage is the variable, the competence of civil courts is the central issue. Courts are involved because the obligations of the insurer are not self-enforcing. When a claim is made, the interests of client and insurer are opposed, and as a consequence common knowledge between the client and the insurer is not sufficient to produce agreement about the claim. Thus insurance contracts are written for variables that can be determined by the courts at the time the contract needs to be enforced.

Damage is, by definition, the cost of restoring the property to its previous condition. The court is competent to determine damage as long as evidence of damage is easy to produce and relatively unequivocal. When the property is restored, the actual costs of restoration are evidence of the damage. Issues can be raised concerning the exact quality of the restoration, but they are not significant in the present analysis. Damage is easily determined as long as the property is restored, and the enforceability of contracts written for damage is the reason that damage is the basis for property insurance indemnities. Damage is ex ante contractible because it is ex post enforceable – part of the time.

Enforcement of the amount of damage is a choice of the client, and it can be a costly one. In order to obtain enforcement from the court, the client restores the property to its previous condition, which is not always the highest valued use of the land. Thus the cost of enforcement is the value of the lost opportunity to convert. When damage is heavy, the highest valued use is typically a conversion and the value of conversion rises as damage increases. At some point the client abandons restoration, foregoes enforcement by the courts, and settles the claim by bargaining with the insurer.

In order to get at the consequences, assume that the client and the insurer know the facts of the case – t, v, l, and q – as common knowledge. Such common knowledge does not imply that contracts can be enforced because the court remains outside the common-knowledge perimeter. The client can declare a higher-than-true value of t, or the insurer can declare a lower-than-true value, leading to an impasse that can be broken either by the court in case the property is restored, or by a bargain between the client and the insurer in case it is converted.

In reality, the client and the insurer might have different information, but whether there is a net advantage either way is unclear. After a property is damaged, both parties spend resources gathering information. Perhaps the client has special expertise in the idiosyncrasies of the property and of the local property market, but she almost surely lacks experience in the situation. The insurer may not have the local information, but it has the advantage in confronting such situations repeatedly. The net advantage could lie with either party. For purposes of analysis, the assumption of common knowledge between the client and the insurer is the right one.

4 Bargaining

After damage t occurs, the insurer and the client together command a certain amount of resources, and the amount depends upon whether the property is restored or converted. First consider restoration. The wealth of the insurer, excluding the premium for this insurance, is W. The wealth of the client before paying the premium is the value of the restored property v and thus the aggregate wealth is $W + v - t$. Other wealth of the client is ignored here without loss of generality. Wealth of the insurer after bargaining is w_1, and wealth of the client is w. Then the wealth constraint is

$$w_1 + w = W + v - t \tag{1}$$

Utility of the insurer is $u_1(w_1)$. Utility of the client is $u = u(w)$. The client is risk-averse. The insurer is risk-averse or risk-neutral. The utility possibility frontier conditional on restoration is given by

$$u = u(W + v - t - u_1^{-1}(u_1)) \tag{2}$$

This curve is the lower one in Figure 1. It is notable mainly for containing the threat point.

The other curve in Figure 1 applies when the property is converted to the best use. The new value of the property is v^*, the cost of getting that value is c^*, the value of converting is $v^* - c^*$, and $v^* - c^*$ is also the value of the land, l. Then the wealth constraint for the case of conversion is

$$w_1 + w = W + l \tag{3}$$

and the utility possibility frontier conditional upon conversion is

$$u = u(W + l - u_1^{-1}(u_1)) \tag{4}$$

That defines the upper curve in Figure 1.

Bargaining occurs when conversion is worthwhile, that is, when $l > v - t$, or, equivalently, when $t > q$, and that situation is illustrated in Figure 1. On the horizontal axis, the distance between the frontiers is $u_1(W + l) - u_1(W + v - t)$ and remembering that $l = v - q$, the distance is $u_1(W + v - q) - u_1(W + v - t)$. Thus $t - q$ is the value of the surplus generated by converting the property. In the general case surplus is $\max[t - q, 0]$.

4.1 Threat point

The case in which damage is below the threshold is not interesting here because the client chooses restoration and as a by-product produces the needed evidence of the amount of damage. In the interesting situation, damage exceeds the threshold, conversion generates a surplus, and the indemnity paid by the insurer is an unknown quantity over which the client and insurer bargain.

The outcome if bargaining fails is the threat point. It is identified here with taking the claim to court and arriving at a particular outcome of the potential court battle. Instead of fighting for the right to convert, the client restores the property and thus accumulates evidence of the amount of damage t. On the basis of this evidence she wins a judgment of $\min[t, b]$. Seeing this, the insurer abandons a courtroom defense and pays the indemnity. Thus the cost of a court battle is avoided. If the client carries out the threat, the insurer's utility is $u_1(W + P - \min[t, b])$ and the client's utility is $u(v - P - t + \min[t, b])$ which are shown as the point T in Figure 1. The threat point is inefficient and undesired by either party.

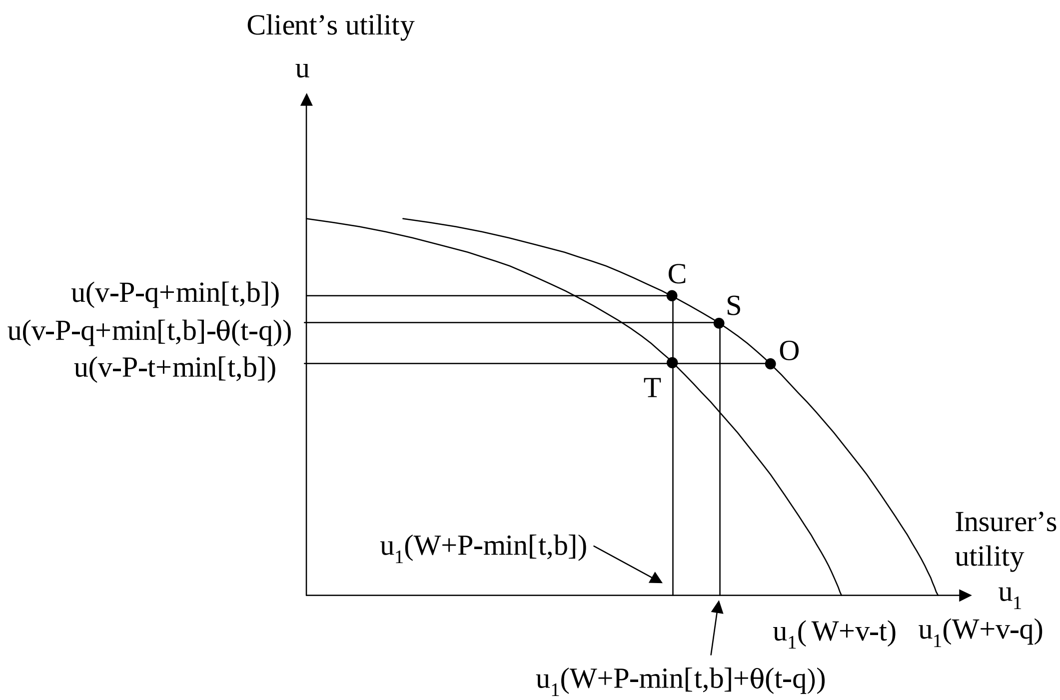

Figure 1: Bargaining for indemnity

The insurer knows the same things that the client knows – damage, conversion threshold, and upper limit. Its threat is similar to the client's. By refusing to settle at the commonly known value t, it forces the client into court. There the client reduces costs by restoring the property and authenticating her claim to an indemnity of $\min[t, b]$ by presenting receipts in the amount t. Again the client gets the utility $u(v - P - t + \min[t, b])$, and the insurer keeps $W + P - \min[t, b]$. Again, the threat point is T in Figure 1. The threat point is not desired by either party because it wastes the conversion surplus, but it has an important influence on the eventual bargain.

Penalties for conversion are implicit in the bargain. The insurer might offer the point labeled O on the utility possibility frontier in Figure 1. The indemnity at O is $\min[t, b] - (t - q)$, giving the client the freedom to convert and keeping all of the conversion surplus for the insurer. The offer implies a penalty for conversion equal to $t - q$. At this point it is necessary for the insurer to supply some vanishingly small reward to the client for taking the trouble to convert. A possible counteroffer of the client is the point labelled C on the utility possibility frontier in Figure 1. At C the indemnity is $\min[t, b]$, and the client keeps the entire surplus from conversion. It is the only point at which the client receives the indemnity $\min[t, b]$ that a naive client believes is promised in the contract. Alternatively, the sophisticated client may feel that the valuable conversion opportunity is the result of her clever property selection and patient risk-bearing. The difference between the offer points is the assignment of the surplus. These considerations define the bargaining problem.

Neither extreme is a likely solution of the bargain. A more reasonable point is one like S in Figure 1 that divides the surplus. At S the client converts the property but sacrifices a portion of the conversion surplus as the price of doing so.

4.2 Constant shares

Theory offers a variety of solutions of the bargaining problem. Nash's solution is the most famous, the generalized Nash is a useful tool of analysis, and these solutions are considered in section 5. Other solutions were not examined in preparing this paper, and they might yield further interesting results.

All solutions considered here are Paretian in the sense that they pick out points such as S on the curve between O and C in Figure 1. For purposes of introducing the main concepts, the constant-shares solution is convenient. In it, the power of the insurer relative to the client is characterized by the parameter θ, which represents the fraction of the surplus gained by the insurer. Under the constant-shares solution, the penalty for conversion is always $\theta \max[t - q, 0]$ and the indemnity is

$$I = \min[b, t] - \theta \max[t - q, 0] \tag{5}$$

The constant shares solution can be justified as a case of the generalized Nash bargaining solution when utility functions display constant relative risk aversion. Denote the penalty for conversion by $z = \theta \min[t-q, 0]$. Suppose that the insurer's net utility gain from the bargain

$$u_1(W + P - \min[t, b] + z) - u_1(W + P - \min[t, b]) \tag{6}$$

is the utility function $\frac{z^{1-r_1}}{1-r_1}$ where $0 < r_1 < 1$. The utility has relative risk aversion constantly equal to r_1. Suppose further that the client's net utility gain from the bargain

$$u(v - P - q + \min[t, b] - z) - u(v - P - t + \min[t, b]) \tag{7}$$

is the utility function $\frac{(t-q-z)^{1-r}}{1-r}$ where $0 < r < 1$, a CRRA utility function with parameter r. Let the solution concept be the generalized Nash bargaining solution with parameter γ. That is, the solution is the z that maximizes

$$\left(\frac{z^{1-r_1}}{1-r_1}\right)^{\gamma} \left(\frac{(t-q-z)^{1-r}}{1-r}\right)^{1-\gamma} \tag{8}$$

The solution satisfies

$$\frac{\gamma(1-r_1)}{(1-\gamma)(1-r)} = \frac{z}{t-q-z} = \frac{\theta}{1-\theta} \tag{9}$$

The share of the insurer varies positively with the parameter γ, as would be expected. Greater risk aversion of the client also raises the insurer's share, and risk aversion of the insurer lowers it. The resulting parameter θ applies regardless of the size of the surplus or the initial wealth positions of the bargainers.

Of course the representation of utility is confining. It does not give scope for variations in v, W, and P to influence the bargain, and it restricts the effects of t and q. These issues are examined in section 5, but in spite of limitations, the constant-shares solution is a reasonable one.

4.3 Insuring damage

The effects of bargaining are mixed. They sometimes improve the client's risk posture and sometimes make it worse. In the area of insuring damage, bargaining compromises the goals of insurance by producing new risks. Consider the textbook case in which insurance is fairly priced and the indemnity is equal to damage up to the insurable interest. Here the insurable interest is the conversion threshold q, and the insured

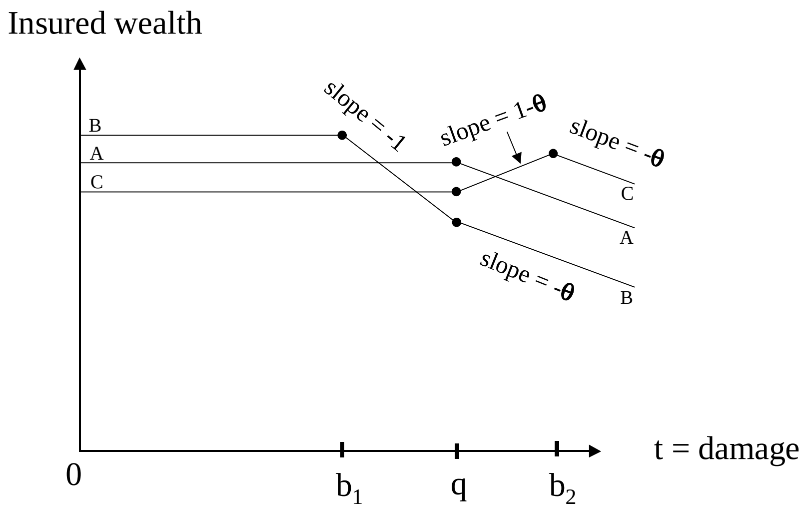

Figure 2: Insured wealth under bargaining: Three choices of upper limit.

wealth of the client should be a level line. In Figure 2 it would lie just below the level part of the AA line.

In the bargaining regime, however, the indemnity is

$$I = \min[b, t] - \theta \max[t - q, 0] \tag{10}$$

and insured wealth of the client is

$$v - P - \min[t, q] + \min[t, b] - \theta \max[t - q, 0] \tag{11}$$

The expression is difficult because of the minimum and maximum functions, but Figure 2 shows the three situations that occur. The path AA represents the insured wealth for a consumer who has by good chance selected an upper limit equal to the conversion threshold, that is, $b = q$. She is following as closely as possible the textbook recommendation by insuring to value, but she does not receive the textbook indemnity. Once damage exceeds the upper limit of coverage, the bargain turns against her. Indemnity and insured wealth fall with rising damage.

The effect of rising damage and falling indemnity is not too paradoxical. When damage is already equal to the upper limit of coverage, further damage reduces the utility of the threat point to the client. The insurer's utility in the threat point is unchanged. See this in Figure 1 by visualizing the shift of the lower curve to the left as damage increases. The vertical line from $W + P - \min[t, b]$ is unmoving because t has exceeded b. The threat point moves straight downward and the upper curve stays in place. The client has a worse bargaining position.

A consumer might try to avoid the problem by choosing a lower upper limit like b_1 in Figure 2. Then her insured wealth is shown by the path BB. Her wealth falls with slope -1 when the upper limit is passed, and continues to fall with slope $-\theta$ after the threshold is also passed. Alternatively, she might try a higher upper limit like b_2 with the result CC in Figure 2. On CC wealth rises with slope $1-\theta$ when damage passes the threshold and falls with slope $-\theta$ when it also passes the upper limit of coverage. Thus bargaining introduces a new risk where none existed before.

Under bargaining, the available paths of insured wealth are all undesirable, with one exception. When the client has all the bargaining power, $\theta = 0$, the right-hand portions of the paths all become level lines. The risk averse consumer chooses an upper limit equal to the threshold and obtains the path BB, which is completely flat. The other paths are rejected at fair prices because they produce under-insurance or over-insurance. The advantages of the solution depend on the upper limit's being equal to the threshold. In reality the threshold varies randomly over time and is known only approximately at any one time. Consequently the $\theta = 0$ case is not so ideal, since the client doesn't have the knowledge of the threshold that would be needed to take full advantage. That problem would be surmounted by a contract in which the threshold is a contractible variable, but in the cases studied here the threshold is not contractible.

Another interesting case is that of the all-powerful adjuster, $\theta = 1$. It fits well with the textbook treatment of insurable interest. In fact, that case supplies optimum insurance at fair prices provided that there is no upper limit on coverage. The presence of an upper limit, even the correct upper limit, puts the indemnities back in the configurations of Figure 2. Since insurance of real estate has upper limits, adjusters are not all-powerful in practice. The case of the all-powerful adjuster and the all-powerful client suggest that varying the bargaining parameter is not a means of making insurance more efficient. It is a means of transforming the inefficiencies from one form to another.

4.4 Choice of the upper limit

Whether rationalized by theory or not, upper limits are the key feature of property insurance. The rational client knows that the indemnity falls as damage rises beyond the upper limit and that her choices are those illustrated in Figure 2. Understanding the situation, she reconsiders the choice of the upper limit. Reducing the upper limit weakens her bargaining position. Raising the upper limit improves it. It seems that the client should select an upper limit higher than the conversion threshold, and this section shows that is correct.

To begin, the fair premium function is

$$P(b) = \int_{t=0}^{\infty} (\min[b,t] - \theta \min[t-q,0])h(t)dt \tag{12}$$

Expected utility is

$$T(b;\theta) = \int_{t=0}^{\infty} u(\overline{v} - P(b) + \min[b,t] - \theta \min[t-q,0] - \min[\overline{q},t])h(t)dt \tag{13}$$

The function is not globally concave in b because the negative of the premium function $-P(b)$ is convex. Nevertheless, start with the first derivative which may be written as

$$T_b(b;\theta) = P'(b)\left[E[u'|t>b] - Eu'\right] \tag{14}$$

The terms in square brackets need explanation. The term Eu' is the marginal utility of certainty wealth spread across all states

$$Eu' =$$

$$\int_{t=0}^{\infty} u'(v - P(b) + \min[b,t] - \theta \min[t-q,0] - \min[t,q])h(t)dt \tag{15}$$

The term $E[u'|t>b]$ is derived using that fact that

$$P'(b) = \int_{t=b}^{\infty} h(t)dt \tag{16}$$

which implies that the conditional probability of t, given $t > b$, is $\frac{h(t)}{P'(b)}$. Then the term is

$$E[u'|t>b] =$$

$$\int_{t=b}^{\infty} [u'(v - P(b) + \min[b,t] - \theta \min[t-q,0] - \min[t,q]] \frac{h(t)}{P'(b)} dt \tag{17}$$

Compared to Eu', this is the marginal utility of the same unit of certainty wealth when it purchases, at fair prices, a uniform increase in wealth in states in which damage exceeds the upper limit of coverage.

Another important relation exists between Eu' and $E[u'|t > b]$. Each is the average of marginal utility over a domain of values of t. With that in mind, the relation of these quantities is shown in Figure 2. Two values of the upper limit are illustrated, b_1 and b_2. Consider b_1. The marginal utilities in $E[u'|t > b_1]$ are integrated over the two segments to the right of $t = b_1$, and those in Eu' are integrated over all three segments. On the other hand, at b_2 the marginal utilities in $E[u'|t > b_2]$ are integrated over only the segment furthest to the right. Again, Eu' aggregates marginal utilities over all three segments.

Consider any upper limit that is, like b_1 in Figure 2, less than the threshold. For such an upper limit, the first term in square brackets in equation (14) is larger than the second because all of the wealths involved are lower, and therefore, by inspection of the diagram, the slope of the objective is strictly positive at points of $(0, q)$. That result holds for all admissible values of the bargaining parameter θ. Thus regardless of the bargain, the optimum is no less than the threshold.

With that preparation, it can be shown that an increase in θ increases the optimum upper limit. Take the particular point $\theta = 0$, and look at the upper limit b_2 in Figure 2. Referring to the derivative of the objective, it is clear that the first term in brackets is dominated by the second term and therefore the derivative is negative for all upper limits that are, like b_2, greater than the threshold. It follows that the optimum at $\theta = 0$ is for the upper limit to be equal to the threshold.

Now what happens when θ rises above zero. It is apparent from the diagram that an upper limit at the threshold no longer equates Eu' and $E[u'|t > b]$. In particular, wealths in the $t > b$ domain are lower than before and consequently $E[u'|t > b]$ is larger than Eu'. This local argument shows that for all $\theta > 0$, the optimum upper limit lies above the threshold q. Global conclusions are difficult to derive because of the unknown size of the neighborhood of concavity around the point $b = q$. In practical cases it is probably quite large.

The same point is established algebraically by looking at the derivatives of Eu' and $E[u'|t > b]$ with respect to θ. Specifically, one wants to fill in the parts of the formula

$$\frac{db}{d\theta} = -\frac{\frac{\partial Eu'|t>b}{\partial \theta} - \frac{\partial Eu'}{\partial \theta}}{\frac{\partial Eu'|t>b}{\partial b} - \frac{\partial Eu'}{\partial b}} \tag{18}$$

$$\frac{dEu'}{d\theta} =$$

$$\int_{t=q}^{\infty} (t-q)u''(v - P(b) + \min[b,t] - \theta \min[t-q,0] - \min[t,q])h(t)dt \tag{19}$$

and

$$\frac{dE[u'|t>b]}{d\theta} =$$

$$\int_{t=q}^{\infty} (t-q)\left[u''(v - P(b) + \min[b,t] - \theta \min[t-q,0] - \min[t,q]\right] \frac{h(t)}{P'(b)} dt \tag{20}$$

Clearly both terms in the numerator of equation (18) are negative and the second term is larger in absolute value. Thus the numerator is negative in equation (18).

Continuing along the same lines,

$$\frac{dEu'}{db} =$$

$$\begin{aligned} &-P'(b) \int_{t=0}^{\infty} u''(v - P(b) + \min[b,t] - \theta \min[t-q,0] - \min[t,q]) h(t) dt \\ &+ \int_{t=b}^{\infty} u''(v - P(b) + \min[b,t] - \theta \min[t-q,0] - \min[t,q]) h(t) dt \end{aligned} \tag{21}$$

$$\frac{dE[u'|t>b]}{db} =$$

$$\begin{aligned} &-P'(b) \int_{t=b}^{\infty} \left[u''(v - P(b) + \min[b,t] - \theta \min[t-q,0] - \min[t,q]\right] \frac{h(t)}{P'(b)} dt \\ &+ \int_{t=b}^{\infty} u''(v - P(b) + \min[b,t] - \theta \min[t-q,0] - \min[t,q]) \frac{h(t)}{P'(b)} dt \\ &- \frac{P''(b)}{P'(b)^2} \int_{t=b}^{\infty} u'(v - P(b) + \min[b,t] - \theta \min[t-q,0] - \min[t,q]) h(t) \end{aligned} \tag{22}$$

Thus the denominator is equation (18) is positive and as a consequence,

$$\frac{db}{d\theta} > 0 \tag{23}$$

The derivation shows that the client selects above all others a path like CC in Figure 2. Although wealth is variable on CC, it varies less widely than on the other paths, and the risk averse client prefers the less variable profile. She raises the upper limit to its optimum level which is above the threshold. Then indemnities rise as damage rises in the interval between the threshold and the upper limit, but economic loss has ceased in this range, so wealth increases. At damage levels above the upper limit, indemnities fall due to bargaining losses. The result is a decrease in the dispersion of insured wealth. Risk aversion is the reason for preferring CC to AA or BB.

The optimum upper limit creates new incentives. The client has more wealth in some of the domain above the threshold than she does in states with less or more damage. A sufficiently skilled arsonist might take advantage of the situation by creating damage in an interval near the upper limit but not too far above or below it. In particular, the highest destruction states are unprofitable. Not too many arsonists can work with sufficient precision. The margin of profit is also thin, apparently a small fraction of the whole value of the property.

The derivation takes the threshold to be deterministic and known to the client but not contractible. The situation is not completely satisfactory because behind the notion of non contractibility is the idea that the threshold varies randomly, and that idea is not used in the derivation.

4.5 Conversion risk and the principle of indemnity

Conversion risk is uncertainty about the threshold. It is a risk inherent in property ownership. Insurance contracts do not address conversion risk explicitly. Nevertheless, insurance can reduce the conversion risk. In a standard textbook on insurance one reads that an indemnity can never exceed the market value of the thing insured. That proposition is called the principle of indemnity. It implies some responsiveness of indemnities to variations in the threshold. In fact, when the threshold decreases the indemnity should decrease, and similarly for increases.

The principle of indemnity is based on decisions of courts. In this paper courts influence indemnities only through their role at the threat point. The principle of indemnity is therefore not a suitable starting point. Under bargaining, the conversion threshold determines the size of the surplus being divided, the threat point, and the eventual indemnity. Assuming that damage and the upper limit exceed the threshold, from equation (5)

$$\frac{dI}{dq} = \theta \tag{24}$$

When the insurer has all the bargaining power, $\theta = 1$ and the outcome is the same as under the principle of indemnity. Less bargaining power of the insurer diminishes the similarity.

The principle of indemnity and the finding that $\frac{dI}{dq} = \theta$ might mean that conversion risk is insured, if the consumer desires to insure it. A rise in the threshold could mean that conversion options and therefore the land have become less valuable. Such an increase in the threshold is a windfall loss that makes the client poorer and is a suitable subject for insurance. The client hopes for insurance against such a windfall, and she receives some through either the principle of indemnity or the bargaining process.

Alternatively, a rise in the threshold can mean that the incumbent structure has become more valuable while conversion options and land value l remain as they were. Such an increase in threshold is a windfall gain that makes the client richer. In this case the principle of indemnity and the bargaining process increase the variability of the consumer's wealth. In summary, the indemnity under bargaining tends to insure variation in land value and to aggravate variations in the whole value of the property.

4.6 Equity Risk

Equity risk is variability in the whole value of the property. It is not addressed by insurance contracts. As with conversion risk, two cases can be distinguished. A drop in v can occur through a drop in land value. Such variation leaves the conversion surplus and the indemnity unchanged in the constant-shares solution. Similarly, the principle of indemnity implies a zero response. Alternatively, v can fall because q falls. In that case the loss of value leads to a rise in the conversion surplus. Under bargaining or the principle of indemnity, the client receives a lower payment, and consequently the bargaining process tends to aggravate the risk. The significant point to be drawn from consideration of conversion risk and equity risk is that insurance indemnities respond to a number of variables not in the contract and the response sometimes tends to make insurance more nearly complete and sometimes to aggravate its incompleteness.

5 Nash bargaining solution

The purpose of this section is to study the penalty for conversion under the Nash bargaining solution and the generalized Nash solution. It is significant because the shares that were constant in the previous section might vary as the other parameters change. The results here predominantly confirm the findings of the previous section, or amend them in minor ways, and they supply answers to questions that could not be approached in the constant-shares solution.

The properties of the Nash solution are sometimes regarded as being valuable mainly for normative reasons, but they also form a useful starting point for descriptive research. On the practical side, the Nash solution equalizes bargaining strength, and that is appropriate in an descriptive study of bargaining. Empirical research would be needed to show that one or the other party is the strongest bargainer. It would be wrong to start with a presumption of greater strength on either side. More deeply, an important element of bargaining literature advances the Nash program of which the objective is a non-cooperative implementation of the Nash solution. In all, the Nash solution is a useful tool for descriptive study of bargaining.

The important properties of the Nash bargaining solution are these: It is unchanged by additions to the utility indicators or multiplications of them. It treats symmetric bargainers symmetrically. It is independent of irrelevant alternatives, it is Pareto optimum, and it is the unique solution possessing all these properties. In addition, it is generated as the solution to a maximization of the product of the net utilities. That is, subtract the utility of the threat point from each bargainers utility function, and then maximize the product of the result,

$$(u_1 - u_1^*)(u - u^*) \tag{25}$$

Some properties of the bargaining indemnities should survive when the balance of bargaining power shifts. To confirm that proposition, the generalized Nash solution is useful. It uses a parameter γ to represent the strength of the insurer and $1 - \gamma$ for the strength of the client. The generalized Nash solution is the one that maximizes

$$(u_1 - u_1^*)^\gamma (u - u^*)^{1-\gamma} \tag{26}$$

The Nash solution is reached in the case of $\gamma = 1/2$. It is important to show that the main results hold up for various values of γ. In this endeavor, it is convenient to divide the exposition in two parts corresponding to whether damage exceeds the upper limit.

5.1 Damage below the upper limit

As before, bargaining is concerned with damage beyond the conversion threshold, that is, $t > q$. Otherwise there is no surplus over which to bargain. Within this domain, consider those bargains in which the upper limit specified in the contract is not yet binding. Let w_1 and w be the wealths of the insurer and the client. On the boundary of the utility possibility set, the wealth constraint is:

$$W + l = w_1 + w \tag{27}$$

The threat point when the upper limit is not binding, $t < b$ is

$$w_1^* = W + P - t \tag{28}$$

and

$$w^* = v - P \tag{29}$$

The interesting variable is the penalty for conversion, denoted by z. It is the amount by which the actual indemnity falls short of the theoretical indemnity t. Thus the insurer moves from $W+P-t$ to $W+P-T+z$ and the client falls from $v-P-q+t$ which would be received if all damage were indemnified to $v-P-q+t-z$ which holds after the penalty for conversion is applied. Using that notation,the problem that generates the Nash bargaining equilibrium is

$$\begin{aligned}&(u_1(W+P-t+z)-u_1(W+P-t))^{\gamma}\\ &\times(u(v-P-q+t-z)-u(v-P))^{1-\gamma}\end{aligned} \tag{30}$$

In order to organize the derivations, regard the maximand as $F(z;t,q,v,W,P)$. The objective is concave because

$$F_{zz} < 0 \tag{31}$$

as shown below. Therefore the bargaining solution $z(t,q,v,W,P)$ is the unique solution to the first-order condition

$$F_z = 0 \tag{32}$$

The first-order condition is

$$\begin{aligned}&F_z = \\ &\gamma\frac{u_1'(W+P-t+z)}{u_1(W+P-t+z)-u_1(W+P-t)}\\ &-(1-\gamma)\frac{u'(v-P-q+t-z)}{u(v-P-q+t-z)-u(v-P)}\end{aligned} \tag{33}$$

The bargaining surplus disappears when damage is less than or equal to the threshold, and the penalty for conversion should disappear as well. In confirmation, the solution function satisfies $z(t,t,v,W,P) = 0$, which can be seen by substituting in F_z at equation (33).

Differentiating again

$$F_{zz} =$$
$$\gamma\frac{\left[\begin{array}{c} u_1''(W+P-t+z)(u_1(W+P-t+z)-u_1(W+P-t)) \\ -(u_1'(W+P-t+z))^2 \end{array}\right]}{(u_1(W+P-t+z)-u_1(W+P-t))^2}$$
$$+(1-\gamma)\frac{\left[\begin{array}{c} u''(v-P-q+t-z)(u(v-P-q+t-z)-u(v-P)) \\ -(u'(v-P-q+t-z))^2 \end{array}\right]}{(u(v-P-q+t-z)-u(v-P))^2} \quad (34)$$

Close inspection shows that F_{zz} is negative for risk-averse or risk-neutral bargainers. Comparative statics

The comparative statics consist of filling out F_{zt}, F_{zq}, F_{zv}, F_{zW}, and F_{zP} in the following template:

$$\begin{aligned} dz &= \frac{\partial z}{\partial t}dt + \frac{\partial z}{\partial q}dq + \frac{\partial z}{\partial v}dv + \frac{\partial z}{\partial W}dW + \frac{\partial z}{\partial P}dP \\ &= \frac{F_{zt}}{-F_{zz}}dt + \frac{F_{zq}}{-F_{zz}}dq + \frac{F_{zv}}{-F_{zz}}dv + \frac{F_{zW}}{-F_{zz}}dW + \frac{F_{zP}}{-F_{zz}}dP \end{aligned} \quad (35)$$

In this setup, the sigh of $\frac{\partial z}{\partial t}$ is the same as the sign of F_{zt}, and the relation holds for the other variables.

The remainder of this subsection is devoted to proving the following proposition.

Proposition 1 Assume that client and insurer possess DARA or CARA utility functions. Let $z(t,q,v,W,P)$ be the penalty and $I(t,q,v,W,P)$ the indemnity. Then when the insurer is CARA or risk-neutral

$$z_t(t,q,v,W,P) > 0 \quad (36)$$

and

$$I_t(t,q,v,W,P) < 1 \quad (37)$$

For the client,

$$z_q(t,q,v,W,P) < 0 \quad (38)$$

and

$$z_t(t,q,v,W,P) + z_q(t,q,v,W,P) \leq 0 \quad (39)$$

with strict negativity if and only if the insurer possesses DARA utility. Further,

$$z_v(t, q, v, W, P) \leq 0 \tag{40}$$

with strict negativity if and only if the client possesses DARA utility,

$$z_W(t, q, v, W, P) \geq 0 \tag{41}$$

with strict positivity if and only if the insurer possesses DARA utility, and

$$z_P(t, q, v, W, P) \geq 0 \tag{42}$$

with strict positivity if and only if at least one actor possesses DARA utility.

The first four parts of the proposition correspond to findings in the constant-shares solution. The last three concern questions that were not approachable in that context. In the constant shares solution the penalty is linear and increasing in damage. Here $z_t(t, q, v, W, P) > 0$ shows that indemnity still increases with damage. It is almost surely not linear. The finding of the previous section that, in the interesting range, indemnity rises more slowly than damage is confirmed by the second result, i.e., that $I_t(t, q, v, W, P) < 1$. The result that

$$z_q(t, q, v, W, P) < 0 \tag{43}$$

shows that for fixed damage, a reduced conversion surplus reduces the penalty, confirming the similar finding of the constant-shares solution. The finding

$$z_t(t, q, v, W, P) + z_q(t, q, v, W, P) \leq 0 \tag{44}$$

shows that when the conversion surplus is unchanged, an increase in damage either reduces the penalty or leaves it unchanged. The proof shows that the rate of reduction would be nil if the insurer is risk neutral and suggests that it is small if the insurer is nearly risk neutral.

The other findings of the proposition have no counterparts in the constant-shares solution. The result that $z_v(t, q, v, W, P) \leq 0$ shows that increased wealth of the client is, if anything, an advantage in the bargain, and $z_W(t, q, v, W, P) \geq 0$ shows that increase wealth of the insurer is, if anything, a disadvantage. The finding that $z_P(t, q, v, W, P) \geq 0$ comes about because the effects of wealth changes are cumulative when wealth is shifted from the client to the insurer by increasing the premium.

The remainder of this subsection derives the results and occasionally discusses them further. Pursue comparative statics first for t through

$$F_{zt} =$$

$$\gamma \frac{\left[\begin{array}{c} -u_1''(W+P-t+z)(u_1(W+P-t+z)-u_1(W+P-t)) \\ +u_1'(W+P-t+z))(u_1'(W+P-t+z)-u_1'(W+P-t)) \end{array}\right]}{(u_1(W+P-t+z)-u_1(W+P-t))^2}$$

$$+(1-\gamma)\frac{\left[\begin{array}{c} -u''(v-P-q+t-z)(u(v-P-q+t-z)-u(v-P)) \\ +(u'(v-P-q+t-z))^2 \end{array}\right]}{(u(v-P-q+t-z)-u(v-P))^2} \tag{45}$$

Suppose that the insurer is risk-neutral. Then the coefficient of γ is zero and all the terms in the coefficient of $1-\gamma$ are positive. Consequently $F_{zt} > 0$, and

$$z_t(t,q,v,W,P) > 0 \tag{46}$$

The same holds true whenever the utility of the insurer displays constant absolute risk aversion. Under CARA, the coefficient of γ is zero. To show the result, compress notation by letting the absolute risk aversions of the actors be denoted by

$$A = -\frac{u''(v-P-q+t-z)}{u'(v-P-q+t-z)} \tag{47}$$

and

$$A_1 = -\frac{u_1''(W+P-t+z)}{u_1'(W+P-t+z)} \tag{48}$$

Under CARA, these expressions are both constants. Then the numerator of the coefficient of γ in equation (45) is

$$\begin{aligned} u_1'(W+P-t+z)-u_1'(W+P-t) &= \int_{\varepsilon=0}^{\varepsilon=z} u_1''(W+P-t+\varepsilon)d\varepsilon \\ &= \int_{\varepsilon=0}^{\varepsilon=z} -A_1 u_1'(W+P-t+\varepsilon)d\varepsilon \\ &= -A_1(u_1(W+P-t+z)-u_1(W+P-t)) \end{aligned} \tag{49}$$

In the other term in the numerator of the coefficient of γ in equation (45)

$$-u_1''(W+P-t+z) = A_1 u_1'(W+P-t+z) \tag{50}$$

Therefore the whole numerator of the coefficient of γ in equation (45) is zero. The contributions of the coefficient of $1-\gamma$ are positive, as before, and therefore in the case of a CARA insurer with any degree of risk aversion, $F_{zt} > 0$.

When the insurer is DARA, the equalities in equation (49) become inequalities. The absolute risk aversion that appears under the integral is always above A_1 which is evaluated at $W + P - t + z$, a relatively high level of wealth. Thus

$$u_1'(W + P - t + z) - u_1'(W + P - t) < -A_1(u_1(W + P - t + z) - u_1(W + P - t)) \tag{51}$$

Consequently the coefficient of γ in equation (45) is negative. The coefficient of $1 - \gamma$ remains positive. It is therefore unclear whether the sign of F_{zt} can ever be negative. I have no examples of negative F_{zt} and no proof that F_{zt} is always positive. The result confirms findings of the previous section but perhaps not as strongly as hoped.

Let $I(t, q, v, W, P) = t - z$ represent the indemnity. When the sign of $z_t(t, q, v, W, P)$ is positive, as it is insurers that are either CARA or nearly risk-neutral,

$$\frac{dI(t, q, v, W, P)}{dt} < 1 \tag{52}$$

The result is consistent with the corresponding result of the fixed share solution. Indemnity rises less than damage once the threshold is exceeded.

Continuing

$$F_{zq} = -(1-\gamma)\frac{\left[\begin{array}{c} -u''(v - P - q + t - z)(u(v - P - q + t - z) - u(v - P)) \\ +(u'(v - P - q + t - z))^2 \end{array}\right]}{(u(v - P - q + t - z) - u(v - P))^2} \tag{53}$$

The term is negative and therefore so is $z_q(t, q, v, W, P)$. For a CARA insurer, $F_{zq} = -F_{zt}$ and $z_q(t, q, v, W, P) = -z_t(t, q, v, W, P)$. The meaning is as follows:. Consider an increase in damage that leaves the conversion surplus unchanged, that is, an increase in t with $t - q = a$ where a is a positive constant. For a CARA insurer, the penalty for conversion is unchanged by increased damage. That agrees with the θ-fixed equilibrium finding on the same question. More generally, the insurer might be DARA. Then

$$z_t(t, q, v, W, P) + z_q(t, q, v, W, P) \tag{54}$$

has the sign of

$$\frac{\left[\begin{array}{c} -u_1''(W + P - t + z)(u_1(W + P - t + z) - u_1(W + P - t)) \\ +u_1'(W + P - t + z))(u_1'(W + P - t + z) - u_1'(W + P - t)) \end{array}\right]}{(u_1(W + P - t + z) - u_1(W + P - t))^2} \tag{55}$$

For the CARA insurer, the expression was zero. For a DARA insurer, it is negative by the same argument that applied in the analysis of F_{zt} at equation (45). Then increased damage with unchanged surplus leads to smaller penalty for conversion. In one interesting case, the insurer is nearly risk neutral, and the penalty does not decline by much.

Similarly

$$F_{zv} = -(1-\gamma) \times \frac{\left[\begin{array}{c} u''(v-P-q+t-z)(u(v-P-q+t-z)-u(v-P)) \\ -u'(v-P-q+t-z)(u'(v-P-q+t-z)-u'(v-P)) \end{array}\right]}{(u(v-P-q+t-z)-u(v-P))^2} \tag{56}$$

The expression is zero if the consumer is CARA. If she is DARA, it is negative. That is, the penalty for conversion falls as the wealth of the consumer rises. Thus for CARA and DARA consumers,

$$z_v(t,q,v,W,P) \leq 0 \tag{57}$$

with strict negativity for DARA consumers. More initial wealth for the client lowers the penalty for conversion, perhaps by improving utility in the threat point. The θ-solution has no corresponding prediction.

Turning to the wealth of the insurer,

$$F_{zW} = \gamma \frac{\left[\begin{array}{c} u_1''(W+P-t+z)(u_1(W+P-t+z)-u_1(W+P-t)) \\ -u_1'(W+P-t+z)(u_1'(W+P-t+z)-u_1'(W+P-t)) \end{array}\right]}{(u_1(W+P-t+z)-u_1(W+P-t))^2} \tag{58}$$

For the CARA insurer the term is zero, and for DARA it is positive. Thus

$$z_W(t,q,b,v,W,P) \geq 0 \tag{59}$$

for CARA and DARA insurers, with strict positivity for DARA insurers.

Next consider

$$F_{zP} = \gamma \frac{\left[\begin{array}{c} u_1''(W+P-t+z)(u_1(W+P-t+z)-u_1(W+P-t)) \\ -u_1'(W+P-t+z))(u_1'(W+P-t+z)-u_1'(W+P-t)) \end{array}\right]}{(u_1(W+P-t+z)-u_1(W+P-t))^2}$$
$$-(1-\gamma) \frac{\left[\begin{array}{c} -u''(v-P-q+t-z)(u(v-P-q+t-z)-u(v-P)) \\ +u'(v-P-q+t-z)(u'(v-P-q+t-z)-u'(v-P)) \end{array}\right]}{(u(v-P-q+t-z)-u(v-P))^2} \tag{60}$$

As a check on consistency or derivations, notice that

$$F_{zP} = F_{zW} - F_{zv} \tag{61}$$

The sign of F_{zP} is always non negative. It is strictly positive if at least one of the actors is DARA instead of CARA. The P raise the penalty by two routes: it undermines the utility of the threat point to the client and improves the utility of the threat point for the insurer. If the insurer is CARA or simply risk-neutral, F_{zW} vanishes and F_{zP} is positive. It follows that, for insurer who is risk-neutral or nearly so, $F_{zP} > 0$ and consequently

$$z_P(t, q, b, v, W, P) \geq 0 \tag{62}$$

for CARA and DARA actors, with strict positivity if at least one actor is DARA. Transferring wealth from the client raises the penalty for conversion. The constant-shares model has no corresponding predictions.

5.2 Binding upper limit

The most interesting implications of the fixed–share solution concern indemnities when the upper limit is binding, $b < t$. The case is still damage greater than threshold, $t > q$ and thus a surplus $t - q$ exists over which the parties bargain. The threat point for the insurer is

$$w_1^* = W + P - b \tag{63}$$

and for the client it is

$$w^* = v - P - t + b \tag{64}$$

These outcomes are the result of non cooperation, that is, of a court battle that might become costly. The battle concerns the true amount of damage t. Because the battle can be costly, the client restores the property to its previous use and supplies the receipts to prove the amount of damage. Seeing the situation, the insurer capitulates and pays b, the upper limit of indemnity.

The derivations in this subsection look the same as those in the previous one, but the results differ significantly. The problem that generates the generalized Nash solutions is to maximize

$$(u_1(W + P - b + z) - u_1(W + P - b))^\gamma \times$$
$$(u(v - P - q + b - z) - u(v - P - t + b))^{1-\gamma} \tag{65}$$

Denote the maximand by $G(z; t, q, b, v, W, P)$. Concavity holds because

$$G_{zz} < 0 \tag{66}$$

is demonstrated below. By abuse of notation, the solution will be denoted by $z(t, q, b, v, W, P)$, which has the argument b in addition to the ones appearing in the $z(t, q, v, W, P)$ of the previous subsection. The solution $z(t, q, b, v, W, P)$ is found from the first-order condition

$$G_z = 0 \tag{67}$$

The comparative statics consist of filling out the following template:

$$dz = \frac{G_{zt}}{-G_{zz}}dt + \frac{G_{zq}}{-G_{zz}}dq + \frac{G_{zb}}{-G_{zz}}db + \frac{G_{zv}}{-G_{zz}}dv + \frac{G_{zW}}{-G_{zz}}dW + \frac{G_{zP}}{-G_{zz}}dP \tag{68}$$

As before, the sign of the derivative of z, for instance z_t is the same as that of the corresponding G-term, for instance G_{zt}. The first-order condition is

$$\begin{gathered} G_z = \\ \gamma\frac{u_1'(W + P - b + z)}{u_1(W + P - b + z) - u_1(W + P - b)} \\ -(1-\gamma)\frac{u'(v - P - q + b - z)}{u(v - P - q + b - z) - u(v - P - t + b)} \end{gathered} \tag{69}$$

The bargaining surplus disappears when damage is less than or equal to the threshold, and the penalty for conversion should disappear as well. In confirmation, rewrite the condition at equation (69) by multiplying through by the denominators $u_1(W + P - b + z) - u_1(W + P - b)$ and $u(v - P - q + b - z) - u(v - P - t + b)$. Then the solution condition is satisfied when $q = t$ and $z(t, t, b, v, W, P) = 0$.

Differentiating again

$$\begin{gathered} G_{zz} = \\ \gamma\frac{\left[\begin{array}{c} u_1''(W + P - b + z)(u_1(W + P - b + z) - u_1(W + P - b)) \\ -(u_1'(W + P - b + z))^2 \end{array}\right]}{(u_1(W + P - b + z) - u_1(W + P - b))^2} \\ +(1-\gamma)\frac{\left[\begin{array}{c} u''(v - P - q + b - z)(u(v - P - q + b - z) - u(v - P - t + b)) \\ -(u'(v - P - q + b - z))^2 \end{array}\right]}{(u(v - P - q + b - z) - u(v - P - t + b))^2} \end{gathered} \tag{70}$$

The second derivative is negative by inspection.

Proposition 2 Assume that the client is risk-averse and insurer is risk-averse or risk-neutral. Let $z(t, q, b, v, W, P)$ be the penalty and $I(t, q, b, v, W, P)$ the indemnity. Then

$$z_t(t, q, b, v, W, P) > 0 \tag{71}$$

$$I_t(t, q, b, v, W, P) < 0 \tag{72}$$

$$z_q(t, q, b, v, W, P) < 0 \tag{73}$$

$$z_t(t, q, v, W, P) + z_q(t, q, v, W, P) < 0 \tag{74}$$

and

$$\frac{dI(t, t-a, b, v, W, P)}{dt} > 0 \tag{75}$$

For client and insurer that are CARA or DARA

$$z_b(t, q, b, v, W, P) \leq 0 \tag{76}$$

with strict negativity if and only if at least one actor is DARA,

$$z_v(t, q, b, v, W, P) \leq 0 \tag{77}$$

with strict negativity if and only if the client's utility function is DARA,

$$z_W(t, q, b, v, W, P) \geq 0 \tag{78}$$

and strict positivity holds if and only if the insurer's utility function is DARA,

$$z_P(t, q, b, v, W, P) \geq 0 \tag{79}$$

and strict positivity holds if and only if at least one of the actors is DARA.

In the constant-shares solution the penalty increases with damage at the constant rate θ. linear and increasing in damage. Here $z_t(t, q, b, v, W, P) > 0$ shows that indemnity still increases with damage but probably not at a constant rate. The most intriguing finding of the constant shares solution is that when the upper limit is binding, the indemnity falls as damage increases. That is confirmed by the finding that $I_t(t, q, b, v, W, P) < 0$. The result that

$$z_q(t, q, b, v, W, P) < 0 \tag{80}$$

shows that for fixed damage, a reduced conversion surplus reduces the penalty, confirming the similar finding of the constant-shares solution. The finding

$$z_t(t, q, b, v, W, P) + z_q(t, q, b, v, W, P) < 0 \tag{81}$$

shows that when the conversion surplus is unchanged, an increase in damage reduces the penalty. It implies that the indemnity $I(t, t - a, b, v, W, P) = b - z(t, t - a, b, v, W, P)$ rises with damage for a fixed conversion surplus. The constant shares model predicted no change in indemnity in the same situation. These results are more general than the corresponding ones for generalized Nash solution and non binding upper limit, as recorded in Proposition 1. Those results depended upon CARA or DARA utility. And the results are stronger in the present of the binding upper limit because they are not erased by risk-neutrality or CARA utility of the insurer as they were previously.

The other findings of Proposition 2 are similar to their counterparts in Proposition 1. They concern variations that had no effect in the constant-shares solution. The magnitudes are of course different, but the patterns of signs and the possibility of zero effects when the insurer is risk-neutral are the same as before. The rest of this section proves and sometimes discusses further the results of Proposition 1.

Pursue comparative statics for t through

$$G_{zt} = (1 - \gamma)\frac{u'(v - P - q + b - z)u'(v - P - t + b)}{(u(v - P - q + b - z) - u(v - P - t + b))^2} \tag{82}$$

The term is positive. Therefore $z_t(t, q, b, v, W, P) > 0$. Let $I(t, q, b, v, W, P)$ represent the indemnity. Here $I = b - z$ and because of $z_t(t, q, b, v, W, P) > 0$,

$$\frac{\partial I}{\partial t} = -z_t(t, q, b, v, W, P) < 0 \tag{83}$$

The result is the same as in the constant-share solution: When damage already exceeds the upper limit of coverage, more damage implies less indemnity.

Continuing

$$G_{zq} = -(1 - \gamma)\frac{\left[\begin{array}{c} -u''(v - P - q + b - z)(u(v - P - q + b - z) - u(v - P - t + b)) \\ +(u'(v - P - q + b - z))^2 \end{array}\right]}{(u(v - P - q + b - z) - u(v - P - t + b))^2} \tag{84}$$

The term is negative and therefore

$$z_q(t, q, b, v, W, P) < 0 \tag{85}$$

The penalty for conversion falls as the conversion threshold rises.

Consider an increase in damage that leaves the conversion surplus unchanged at some arbitrary positive level a. As t rises, $q = t - a$ rises by the same amount. The effect on the penalty is

$$\begin{aligned}
\frac{d}{dt} z(t, t-a, b, v, W, P) &= z_t(t, t-a, b, v, W, P) + z_q(t, t-a, b, v, W, P) \\
&= \frac{G_{zt} + G_{zq}}{-G_{zz}} \\
&= \frac{1}{-G_{zz}} \left[-(1-\gamma) \frac{\begin{pmatrix} -u''(v-P-q+b-z) \times \\ (u(v-P-q+b-z) - u(v-P-t+b)) \end{pmatrix}}{(u(v-P-q+b-z) - u(v-P-t+b))^2} \right]
\end{aligned} \tag{86}$$

The expression is negative. Therefore

$$\frac{d}{dt} z(t, t-a, b, v, W, P) < 0 \tag{87}$$

The penalty falls as the amount of damage increases, keeping the conversion surplus constant.

Looking at variation in the upper limit,

$$\begin{aligned}
&G_{zb} = \\
&\gamma \frac{\begin{bmatrix} -u_1''(W+P-b+z)(u_1(W+P-b+z) - u_1(W+P-b)) \\ +u_1'(W+P-b+z)((u_1'(W+P-b+z) - u_1'(W+P-b)) \end{bmatrix}}{(u_1(W+P-b+z) - u_1(W+P-b))^2} \\
&+(1-\gamma) \frac{\begin{bmatrix} -u''(v-P-q+b-z)(u(v-P-q+b-z) - u(v-P-t+b)) \\ +u'(v-P-q+b-z)(u'(v-P-q+b-z) - u'(v-P-t+b)) \end{bmatrix}}{(u(v-P-q+b-z) - u(v-P-t+b))^2}
\end{aligned} \tag{88}$$

This G_{zb} vanishes if both actors have CARA utility. It becomes negative if either is DARA. Thus

$$z_b(t, q, b, v, W, P) \leq 0 \tag{89}$$

for actors that are CARA and DARA, with strict negativity if at least one is DARA. That justifies in the Nash context the finding from the constant-share setting that a higher upper limit improves the bargaining position of the client.

Similarly

$$G_{zv} = +(1-\gamma)\frac{\begin{bmatrix} -u''(v-P-q+b-z)(u(v-P-q+b-z)-u(v-P-t+b)) \\ +u'(v-P-q+b-z)(u'(v-P-q+b-z)-u'(v-P-t+b)) \end{bmatrix}}{(u(v-P-q+b-z)-u(v-P-t+b))^2} \tag{90}$$

Using an argument from the first part of the proof of Proposition 1, the term is zero for clients with CARA utility and negative for DARA utility. Thus

$$z_v(t,q,b,v,W,P) \leq 0 \tag{91}$$

with strict negativity when the client's utility function is DARA. As before when the upper limit was not binding, more initial wealth for the client improves her bargaining position and lowers the penalty for conversion.

The wealth of the insurer acts through

$$G_{zW} = \gamma\frac{\begin{bmatrix} u_1''(W+P-b+z)(u_1(W+P-b+z)-u_1(W+P-b)) \\ -u_1'(W+P-b+z)((u_1'(W+P-b+z)-u_1'(W+P-b)) \end{bmatrix}}{(u_1(W+P-b+z)-u_1(W+P-b))^2} \tag{92}$$

The term is zero when the insurer is CARA and positive when it is DARA.

$$z_W(t,q,b,v,W,P) \geq 0 \tag{93}$$

for insurers that are CARA or DARA, and strict positivity holds when they are DARA. The size of the effect is small if the insurer is nearly risk neutral.

Next consider

$$G_{zP} = \gamma\frac{\begin{bmatrix} u_1''(W+P-b+z)(u_1(W+P-b+z)-u_1(W+P-b)) \\ -u_1'(W+P-b+z)((u_1'(W+P-b+z)-u_1'(W+P-b)) \end{bmatrix}}{(u_1(W+P-b+z)-u_1(W+P-b))^2} +(1-\gamma)\frac{\begin{bmatrix} u''(v-P-q+b-z)(u(v-P-q+b-z)-u(v-P-t+b) \\ )-u'(v-P-q+b-z)(u'(v-P-q+b-z)-u'(v-P-t+b)) \end{bmatrix}}{(u(v-P-q+b-z)-u(v-P-t+b))^2} \tag{94}$$

Noting that

$$G_{zP} = G_{zW} - G_{zv} \tag{95}$$

it follows that for actors who are CARA or DARA,

$$z_P(t, q, b, v, W, P) \geq 0 \tag{96}$$

with strict positivity when at least one is DARA. Transferring wealth from the client to the insurer raises the penalty for conversion.

The derivations confirm the major findings of the analysis using the constant-share solution and produce several implications not available in that setting.

6 Bargaining penalties and contractual penalties.

The present analysis is a complement and an alternative to the contract theory of Bourgeon and Picard. More detail on their theory is needed at this point in order to support the more important discussion of issues that follows. Their paper addresses a problem of over insurance that arises because the conversion threshold is private information of the client while the damage is publicly known. A naive insurance pays damage even when damage is above the threshold, and the client is over insured with some probability. To ameliorate the problem, they study contracts that have two schedules of indemnity, $x(t)$ when the property is restored and a lesser amount $y(t)$ when it is converted. The second-best optimum maximizes over the choice of the functions $x(t)$ and $y(t)$ the expected utility

$$\int_0^\infty \int_{q_0}^{q_1} u(v - P + \max[x(t) - t, y(t) - q]g(q|x)dqh(t)dt \tag{97}$$

with the constraint that P is the fair premium for the indemnities selected. No reason exists to repeat the derivations here. The solutions to the problem turn out to satisfy

$$0 < y(t) < x(t) < t \tag{98}$$

and

$$0 < y'(t) < x'(t) < 1 \tag{99}$$

The finding that $x(t) < t$ means that the client chooses some degree of coinsurance and is natural because over-insurance is endemic. In the contract theory, the penalty (or discount) for conversion is $x(t) - y(t)$ and it rises with rising damage. The penalty or discount is positive because when the client chooses to convert, damage exceeds the loss of wealth. The penalty is greater as damage rises because that entails, in probability, a greater amount of over insurance to be combatted.

By constraint, the penalty is constant independently of the threshold and varies with damage, a configuration that is potentially testable. It contrasts with the property of the bargaining model that the penalty varies with damage and with the threshold and is approximately constant for fixed conversion surplus. The comparison is valuable as a thought experiment, and it might also be the basis for empirical research.

Bourgeon and Picard further discuss the role of arson in the penalty for conversion, but that part of their work is beyond the scope of this paper. Some further comparisons of their model and the present one are discussed below. The above description should be adequate to support the much more important discussion of theoretical issues that follows directly.

6.1 Contractibility theory

The important relations between the contract theory of penalties and the present analysis do not involve derivations. They center on the analysis of contracts. In this regard the papers are close, and a thorough discussion of the similarities is warranted. The discussion here revisits ideas sketched in the introduction, and that too is appropriate.

The Bourgeon and Picard model diverges from textbook ideas of insurable interest. In holding that the threshold is not contractible, it denies that insurable interest is an operative concept in settling insurance claims – in some contexts, at least. Noncontractibility enters because the players in the drama have different information. The client knows the threshold and the insurer doesn't, and therefore the threshold is not a contractible quantity. The denial is important because it recognizes that enforcing contracts is costly. In fact, enforcing contracts that involve the threshold is prohibitively costly. The cost model is a significant advance. The connection between informational asymmetries and non contractibility is similar to that in the literature on principal-agent problems. Here it makes a contribution to a different type of problem.

The Bourgeon-Picard framework is more general than it seems because non contractibility can exist in spite of even common knowledge of the threshold. The client can demand \$250,000 while knowing that the threshold is \$200,000, and the insurer that knows the same truth can offer \$150,000. The client knows that the insurer knows that the value is \$200,000, and the client knows that the insurer knows that the client knows that the insurer knows it, and so on, and also the other way around, but they both deny the truth. Nothing in their power breaks the impasse. Another element is needed, a court in which at reasonable expense some evidence of the true, commonly known value leads to a judgment and a court-ordered settlement. Actual courts are often incapable of making the needed decisions at reasonable cost and consequently non contactibility survives the presence of common knowledge. The contract model of penalties for conversion can flourish even in a common knowledge situation.

This observation significantly reinforces the Bourgeon and Picard results because insurance adjusters are smart, informed, and motivated. Often they are attorneys starting their careers. They are not ignorant of the factors influencing the threshold, and a model resting on their ignorance would be shaky. The contract model ultimately rests on the inability of courts to receive and confirm at reasonable expense the type of information the client and insurer very possibly possess.

Common knowledge is also not necessary for contractibility. The reason lies in the central role played by the court. Perhaps the client is somewhat ignorant of economic opportunities, or perhaps the insurer is acting on behalf of a Fair Access to Insurance plan and has few incentives for careful adjusting. The parties have some knowledge, but not common knowledge and not the same knowledge. The first does not know what the second knows or whether the second knows what he (the first) knows. The court may nevertheless render good judgments without much expense, and the poorly informed parties may nevertheless know enough to assure themselves fair judgments. Because common knowledge is not necessary for contractibility, the contractual model of conversion penalties fails in some situations in which the threshold is private knowledge. Automobile collision insurance, for instance, seems to operate using blue book values to the near-total exclusion of information privately held by either party. Conversion penalties seem not to play a large role and certainly not an explicit one. The penalty for conversion is nevertheless a real phenomenon in many types of property insurance.

A key point in the contractual model of conversion penalties is the concept of damage. Damage means the cost to restore the property and is distinct from 'damages,' a term that stands for the loss of wealth from the damage. Damages are limited by the threshold and damage is not. The distinction is wholly consistent with the approach take by Bourgeon and Picard. To pass blithely from damage to damages without noting the difference would vitiate both the contractual model and the bargaining model[1].

Damage is a contractible quantity in the contractual model. That means a court can value damage accurately and inexpensively. Common knowledge of damage by the client and insurer is not sufficient. In addition, the court must also be inside the common knowledge perimeter.

The bargaining model is an alternative vision. It agrees with its predecessor in most ways. It agrees completely that the threshold is not contractible. It departs on

[1] In an early version of their paper Bourgeon and Picard sometimes use 'damages' to mean damage. That is a slip of the pen and not a lapse in analysis or an exception to the conclusions drawn here.

the contractibility of damage. In looking at damage, the bargaining model considers the position of the court. The court deals in magnitudes that can be testified to. It respects receipts for actual expenses and not speculative valuations or indicators of value from thin markets. When damage is small, the property is restored, and the receipts for costs of restoration are good evidence of the damage. When damage is large, its valuation is almost entirely a speculative valuation and it depends critically upon the value of urban land, for which markets are notoriously thin. Receipts for actual restoration are not available as evidence because the client and the insurer have no desire to restore the property. The court is unable to rule. Thus damage beyond the threshold is a non contractible quantity. Non contractibility of damage beyond the threshold is inconsistent with the model of Bourgeon and Picard because their indemnity functions depend on damage in the range where damage exceeds the threshold. In determining its applicability to a particular situation, the contractibility of damage is a prime concern.

The discussion dwells on contractibility issues because they are subtle and centrally important. There is another way, however, in which the bargaining model departs from the contractual penalties model. In the latter, the interesting action takes place ex ante when insurance is purchased, and the eventual settlement of a claim is a foregone conclusion. The bargaining model views the ex ante purchase of insurance as a preliminary to the negotiations that occur when a claim is settled. The bargaining model poses the question: why should the insurer, or the client, settle for the sums that are supposedly fixed in the contract? The answer is that they shouldn't.

The bargaining approach can even be applied to the optimum penalty contracts. Suppose that damage occurs and by contract a penalty for conversion attaches to it. Suppose further that the client wants to convert. The insurer should declare a lower level of damage. The client should claim a higher amount of damage. When the conflicting claims are settled, it can happen that the contractual penalty prevents a conversion that would otherwise be made. Then the insurer and client should recognize the opportunity for mutual gain and get together in some type of bargain. Thus even the theory of optimum penalties invites further analysis of bargaining.

7 Concluding remarks

One is accustomed to seeing inefficiencies that arise because some markets are missing or because of informational asymmetries. Here the inefficiencies exist in the presence of common knowledge and, seemingly, with trade in all of the relevant markets. According to the bargaining analysis, the legal structure surrounding the contracts is at fault.

The present paper has focused on properties of the bargain. It has not attempted a comprehensive general analysis of the system involving ex ante contracts and ex post negotiations. Nevertheless, the analysis in the fixed-share solution suggests some generalities. It suggests that consumers are best served when the insurer is a strong bargainer. The finding might be valuable to a firm that could lower premiums and adjust more strictly. Whether this can be a valid tactic is a marketing question to which the answer is unclear because price competition in insurance is not always effective.

At the level of the courts, the analysis suggests further attention to their function as a threat point in settlement negotiations, most of which will be settled out of court. As for insurance regulators, the analysis makes an important distinction. Consumers in general benefit from lower rates when insurers are strong bargainers and when each bargain is based upon a given, ethical threat point. Unfair claims practices are actions that move the threat point, that is, they are typically threats and misinformation. The analysis suggests that unethical threats, although seldom acted upon, can have a significantly adverse impact in settlements.

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