Coping with Catastrophic Risk: The Role of (Non)-Participating Contracts

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Abstract

This paper examines how the two fundamental principles in risk allocation, i.e., the mutuality principle and the transfer principle, can be implemented in order to manage efficiently catastrophic risk. From a decomposition of the insurable loss into idiosyncratic and systemic components, we show that the systemic risk is first filtered through a participating policy with a variable premium based on the realized systemic loss, and then it is transferred through an insurance contract providing a coverage on the variable premium. This model is reconsidered to examine the financing of catastrophe risk with alternative risk transfer solutions. Group captives, offering participating policies, are shown to be market enhancing.

Keywords : hedging, insurance, mutuality principle, participating policy, risk transfer, systemic risk.

1. Introduction

The year 2001 was a terrible year for catastrophic events. With $34 billion of insured losses in property insurance, it is comparable with losses in 1992 (hurricane Andrew) and 1999 (windstorms in Europe). While the 1992 and 1999 record losses were due to natural catastrophes, it was the man-made losses that weighted heaviest on the insurers' book in 2001. Property and business interruption losses arising from the terrorist attack of September 11 on New York and Washington are estimated at $19 billion (Swiss Re 2002).[1] Moreover, the forth most costly insurance losses in 2001 were caused by another man-made catastrophe; the explosion in fertilizer factory in Toulouse (France) where insured losses are estimated at $1.4 billion. Even if insurance losses from the September 11 terrorist attack are still difficult to quantify, the resulting total loss is certain to be much larger than that incurred by Hurricane Andrew.

The potential losses from natural catastrophes and, since the September 11 event, man-made catastrophes lead financial researchers to address the financing of catastrophic losses and efficient risk sharing rules between (re)insurance and capital markets. The economic role of insurance and reinsurance companies as private mechanisms of risk allocation is closely related to the concept of insurability. The problem arises because catastrophic risks are not widely diversifiable in an insurance context. In other words, the risk pooling mechanism is not highly efficient in risk reduction. Two types of solutions have been proposed and put into practice in order to fulfill the insurance capacity gap. The first alternative is mandatory public provision of insurance. It relies on the financial ability of the government to spread losses across many citizens. It was imposed in France where the Parliament voted in July 1982 a law instituting a mandatory compensation system for natural disasters. The second alternative is the development of financial instruments to transfer catastrophic risks to financial investors through capital markets. It relies on the huge financial capacity of capital markets. This paper focuses on this second alternative and on its optimal combination with the risk pooling mechanism.

Several studies in the insurance literature have examined the problem of designing optimal risk-sharing contract between a risk-averse insured and a less risk-averse insurer in the presence of a single source of risk (see, e.g., Arrow 1965, Raviv 1979) or when the insured also bears an uninsurable background risk (see, e.g., Gollier 1996, Mahul 2000). When the two sources of uncertainty are insurable, Raviv (1979) shows the optimality of the insurance policy with a single deductible on the aggregate loss. Gollier and Schlesinger (1995) reexamine this problem when insurance markets offer separate contracts for separate (additive) loss exposures and derive the second-best insurance contract design in this incomplete market. These risk-sharing arrangements are based on the transfer principle; risk is transferred to an external risk bearer (e.g., an insurance company or a financial investor) who has a comparative advantage to bear it. As noticed by Doherty and Dionne (1993), the extensive literature on optimal insurance contract design usually ignores a second fundamental principle in the allocation of risk; the mutuality principle. This principle, developed by Borch (1962) and Wilson (1968),

[1] In addition, liability and life insurance losses related to the September 11 event are estimated between $16.5 and 39 billion.

states that the Pareto optimal risk-sharing rule in an economy with risk-averse agents and no transaction costs is such that the final wealth of any agent depends only on the aggregate wealth.

A notable exception is the paper written by Doherty and Dionne (1993). In a four-state expected utility framework, they provide explanations about the proliferation of participating policies provided by mutual insurers and new insurance contract designs from a decomposition of the insured losses into systemic (i.e., undiversifiable) and idiosyncratic (i.e., diversifiable) components.[2] They show that a decomposed risk transfer that efficiently uses this decomposition to apply the two fundamental principles weakly dominates a simple transfer in which the insured loss is not decomposed. Doherty and Schlesinger (2002) reconsider this problem in a framework where consumers have preferences for second-degree stochastic dominance, including the expected utility theory as a particular case. They derive a variable participation insurance policy defined as a convex mixture of a non-participating contract (with a fixed premium) and a fully participating contract (with a variable premium) in which the level of participation is endogenous. However, these two studies are based on two central assumptions. First, coinsurance contracts are only at the policyholder's disposal. Second, the fully participating policy is sold at a fair price; the idiosyncratic component of the individual risk can be managed within this policy at no transaction costs. Such an assumption seems to be difficult to justify as regard real-world insurance markets. The prevalence of transaction costs (e.g., agency costs, administrative costs) in the insurance industry is a well-established fact. For many insurance lines like automobile insurance, transaction costs amount up to 30% of the premium. They would thus prevent insurance companies from offering actuarially fair insurance contracts.

This paper first examines, from a decomposition of the individual loss, the optimal risk-sharing rule between a risk-averse firm and a risk bearer in the multiple state expected utility framework. The originality of our approach stems from the fact that the two above mentioned assumptions are relaxed; risk-sharing contracts are not restricted to coinsurance and the fully participating policy is not necessarily sold at an actuarially fair price. This allows us to show how the cost effectiveness of insurance affects the optimal risk-sharing rule. The catastrophic risk is shown to be filtered through a participating policy with a variable premium based on the realized systemic loss, and then it is transferred through an insurance contract which displays full coverage above a deductible on the variable premium. An optimal variable participating insurance policy is designed from optimal fully participating and non-participating contracts.

The second purpose of this paper is to investigate the financing of catastrophic risk with two alternative transfer solutions; group captives and index-based derivatives. Group captives act as purveyors of fully participating policy and index-based contracts provide coverage on the catastrophic component of the loss. The fully participating policy is shown to be market enhancing. This is illustrated from the analysis of the dominance relationship using the efficient frontier of the combination of two separate

[2] Other explanations have been provided on the role of participating policies offered by mutual insurers and their coexistence with other kinds of organizations. They could help to redress agency problems (Mayers and Smith 1986), adverse selection (Smith and Stutzer 1990) or moral hazard (Smith and Stutzer 1995). Our paper does not deal with such problems of informational asymmetries.

non-participating contracts and that of the variable participating policy in the mean-variance framework.

The remainder of this paper is organized as follows. The model is described in section 2. Optimal fully participating and non-participating insurance contracts are designed in section 3. Section 4 examines the financing of catastrophic risk with alternative risk transfer solutions and shows how group captives can increase efficiency on financial markets. A conclusion highlights the main results and uses our model to investigate optimal risk management of hybrid covers recently offered by insurance and reinsurance companies.

2. The Model

The theoretical analysis is developed into a well-accepted and unified framework; the expected utility theory. A risk-averse firm is endowed with non-random initial wealth w_0 which is subject to a risk of loss $\tilde{y}$, with $0 \le y \le w_0$ for all y.[3,4] This loss can be partitioned into a systemic component $\tilde{x}$ and an idiosyncratic component $\tilde{e}$ through the deterministic relationship

$$(1) \qquad y = l(x,e),$$

with $x \ge 0$, $e \ge 0$, $l_x \ge 0$ and $l_e \ge 0$.[5] The two sources of risk affecting the individual loss are assumed to be stochastically independent. The decomposition (1) is determined according to the risk pool which the firm is part of.[6] For example, in a pool where members are located in a given earthquake-prone area, define the $\tilde{y}$ risk as the individual risk exposure to an earthquake. The $\tilde{x}$ risk would be the common uncertainty component which affects all the members of the pool, i.e., the intensity of the earthquake. The $\tilde{e}$ risk would be the impact of local parameters on the individual losses which can be considered as independent among the members of the risk pool. The second source of risk is (partially) diversifiable within the risk pool, while the first one is undiversifiable, at least at the risk pool's level. However, it may be (partly) swapped with groups of individuals who are exposed to different (and independent) catastrophes or transferred to financial investors with a worldwide diversified business portfolio.

The risk-averse firm can purchase two types of insurance contracts; a non-participating policy and a fully participating policy. The non-participating policy is in the line of standard insurance contracts (see, e.g., Arrow 1965, Raviv 1979). It is described by couple $[J(.,.),Q]$, where Q is the fixed premium and the function $J(x,e)$

[3] The firm can also be assumed to be risk neutral but it is motivated to hedge by market imperfections, direct and indirect costs of financial distress, and convex tax schedule. It thus behaves as if it was risk averse (see Doherty (2000) for a detailed analysis on why risk is costly to firms).
[4] Tildes are used to denote random variables, and the same variables without the tilde denote realizations of the random variables.
[5] A subscript symbol denotes a partial derivative with respect to that symbol.
[6] The size of this pool is assumed to be finite but sufficiently large so that, from the law of large numbers, deviations of average idiosyncratic loss from expected idiosyncratic loss are insignificant.

is the amount of indemnity payments made by the insurer when the realized indices are (x,e). The indemnity schedule is assumed to be nonnegative:

(2) $J(x,e)\geq 0$ for all (x,e),

and the premium is assumed to be a non-decreasing function of the expected indemnity:

(3) $Q=d\left(EJ(\tilde{x},\tilde{e})\right)$,

with $d(0)=0$ and $d'(s)\geq 1$ for all s, and E denotes the expectation operator with respect to the corresponding random variables. The firm can also purchase a fully participating insurance policy. It is described by couple $\left[I(.,.),P(.)\right]$, where the indemnity schedule $I(x,e)$ is assumed to be nonnegative:

(4) $I(x,e)\geq 0$ for all (x,e).

The insurance premium is variable and it depends on the realized systemic component. We assume that it is based on the *ex post* average indemnity paid by the insurer:

(5) $P(x)=c\left(EI(x,\tilde{e})\right)$,

with $c(0)=0$ and $c'(s)\geq 1$ for all s. The premium is thus subject to *ex post* adjustments. For instance, the insured firm may first pay a premium based on the expected value of the systemic risk, $P(E\tilde{x})$. It is then adjusted depending on the realization of the systemic component x; $P(x)<P(E\tilde{x})$ would entail the payment of a dividend to the insured firm while $P(x)>P(E\tilde{x})$ would lead to a premium surcharge paid by the firm.[7]

The problem of the risk-averse firm is to determine the indemnity schedule and the premium of both fully participating and non-participating insurance contracts that maximize its expected utility of final wealth:

(6) $$\max_{I,J,P,Q} Eu\left(w_0-l(\tilde{x},\tilde{e})+I(\tilde{x},\tilde{e})-P(\tilde{x})+J(\tilde{x},\tilde{e})-Q\right)$$
subject to conditions (2) to (5),

where $u(.)$ is a twice-differentiable von Neumann-Morgenstern utility function, with $u'>0$, $u''<0$.

The originality of this paper with respect to previous work (Doherty and Dionne 1993, Doherty and Schlesinger 2002) is twofold. First, we allow the decision-maker to determine the design of optimal non-participating and fully participating insurance contracts, without restricting insurance design to coinsurance. Second, we allow the insurer to sell the fully participating policy at an unfair price.[8]

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[7] Insured firms are assumed to always pay in full *ex post* additional premiums in the event of a large catastrophe. Hence, there is no default risk from the policyholders.

[8] This paper is not a direct extension of Doherty and Schlesinger (2002) because the current problem is investigated in the expected utility framework whereas their analysis is developed under a weaker assumption about preferences under uncertainty, the aversion to mean-preserving spreads.

3. Optimal Insurance Contract Designs

The following proposition shows how the design of optimal fully participating and non-participating contracts is based on the cost effectiveness of insurance.

Proposition 1. *The optimal indemnity schedules of fully participating and non-participating insurance contracts I^* and J^*, solutions to problem (6), take the following form:*

(i) *If $c(s) < d(s)$ for all s, then there exist $D_1 \geq 0$ and $D_2 \geq 0$ such that*
$I^*(x,e) = \max(l(x,e) - D_1, 0)$ *and* $J^*(x,e) = \max(P(x) - D_2, 0)$.

(ii) *If $c(s) > d(s)$ for all s, then there exists $D_3 \geq 0$ such that*
$J^*(x,e) = \max(l(x,e) - D_3, 0)$ *and* $I^*(x,e) \equiv 0$.

The proof is given in the Appendix. Suppose first that the cost of insurance is higher under the non-participating policy than under the fully participating contract, $c(s) < d(s)$ for all s. Such an assumption is justified if both contracts face identical administrative costs but the insurance premium of the non-participating policy is also charged by a risk premium.[9] It can be motivated by the presence of the systemic component which cannot be diversified by the shareholders of the insurance company. As a consequence, they will ask for a risk premium which will increase the cost of capital of the company. This cost will be passed on to the policyholders through a larger premium rate. The optimal insurance strategy as described in Proposition 1(i) is as follows. The firm purchases the fully participating contract in order to (partially) cover the idiosyncratic component of its individual risk exposure. Such a contract provides the firm with full insurance against the $\tilde{e}$ risk above a deductible, while it bears all the systemic risk through the variable insurance premium. In a second step, the non-participating contract is used to provide (partial) coverage only on the $\tilde{x}$ risk through full insurance on the variable insurance premium of the fully participating policy above a straight deductible.

The second part of Proposition 1 is quite intuitive. Suppose the cost of insurance under the fully participating policy is higher than that under the non-participating contract. This could be the case if, because of economies of scale, the administrative costs plus the risk premium of the latter are lower than the administrative costs of the former. The firm will only purchase the non-participating policy displaying full insurance against its individual loss above a straight deductible. This corresponds to the standard result in the insurance literature when the non-participating policy is only available (Arrow 1965, Raviv 1979).

The optimal level of deductibility is characterized in the following proposition.

[9] Given assumption (3) on the insurance premium, this risk premium depends only on the expected indemnity payoff.

Proposition 2. *Suppose that $c(s) < d(s)$ for all s.*

(i) *Under the fully participating policy, the optimal deductible D_1 equals zero if the premium is actuarially fair [$c'(s) = 1$ for all s] whereas it is positive if the premium is unfair [$c'(s) > 1$ for all s].*

(ii) *Under the non-participating policy, the optimal deductible D_2 equals zero if the premium is actuarially fair [$d'(s) = 1$ for all s] whereas it is positive if the premium is unfair [$d'(s) > 1$ for all s].*

The proof is given in the Appendix. The optimal deductible satisfies standard results under both insurance policies, i.e., it equals zero under fair insurance and it is positive when insurance is sold at an unfair price. Under unfair premiums, it should be noticed that the optimal deductible of a contract depends not only on the cost of insurance of this contract but also on that of the other one. If $c(s) > d(s)$ for all s then, from Raviv (1979), the optimal deductible is zero if $d(.) \equiv 0$ and it is positive otherwise.

Propositions 1 and 2 characterize the optimal insurance strategy with fully participating and non-participating contracts. This optimal strategy is illustrated under a multiplicative relationship between the two components of the random individual loss, $l(x,e) = xe$ with $x > 0$. The individual loss is thus proportional to the value of the systemic component. In the case of insurance for natural disasters, $x = 1.5$ would mean that all the individual losses increase by 50 percent because of the occurrence of a catastrophe. In the case of agricultural revenue insurance, the $\tilde{e}$ component could be yield shortfalls caused by a local weather event (in quantity) and the $\tilde{x}$ component could be the crop price at harvest (assumed to be independent with the crop yield risk). While the former can be diversified (if we exclude natural disasters), the latter is not diversifiable. Such an example looks like insurance at replacement cost where a random inflation rate impacts all claims in the same proportion. The optimal participating policy is $I^*(x,e) = \max(xe - D_1, 0)$ and the loss borne by the insured firm after the payment of the indemnity net of the premium is

$$(7) \qquad xe - \max(xe - D_1, 0) + P(x) = x[e - \max(e - D_1/x, 0)] + P(x) = \begin{cases} D_1 + P(x) & \text{if } e \geq D_1/x \\ xe + P(x) & \text{if } e \leq D_1/x. \end{cases}$$

For sufficiently large idiosyncratic losses, $e \geq D_1/x$, the loss borne by the firm is only based on the systemic loss through the variable premium. The systemic risk is thus filtered through the fully participating policy.

Suppose the policy is sold at fair price, as in previous works (Mahul 2001, Doherty and Schlesinger 2002). From Proposition 2, the optimal policy becomes $I^*(x,e) = xe$ and the variable premium is $P(x) = xE\tilde{e}$. As a consequence, the optimal indemnity schedule net of the premium is $I^*(x,e) - P(x) = x(e - E\tilde{e})$ and the loss after net indemnification is $xE\tilde{e}$. The firm is thus fully insured against the idiosyncratic component of its loss, while it is fully exposed to the systemic risk.

The optimal non-participating policy is $J^*(x,e) = E\tilde{e}\max\left(x - D_2/E\tilde{e},0\right)$. The contract provides full insurance on the systemic loss above a positive deductible. Of course, if this contract is sold at a fair price, the firm is fully insured against its loss. This corresponds to the standard result in the literature of insurance economics.

The fully participating and non-participating insurance policies can be combined to construct the so-called *variable participating insurance contract*. In our framework, its indemnity schedule is $V(x,e) = I(x,e) + J(x,e)$ and its premium is $T(x) = Q + P(x)$. We thus allow the firm to choose the form of its participation through the choice of the indemnity schedules of fully participating and non-participating contracts. From the above analysis, the optimal variable participating policy satisfies

(8) $$V^*(x,e) = \max\left(l(x,e) - D_1,0\right) + \max\left[P(x) - D_2,0\right],$$

with $T(x) = P(x) + d\left(E\max\left[P(\tilde{x}) - D_2,0\right]\right)$ where $P(x) = Ec\left(\max\left(l(x,\tilde{e}) - D_1,0\right)\right)$. It should be noticed that this policy differs from the 'variable participation contract' proposed by Doherty and Schlesinger (2002). Beyond the fact that they restrict the insurance contract design to coinsurance, they impose the indemnity of the non-participating policy to depend on both realized idiosyncratic and systemic losses. Such a constraint turns out to be sub-optimal from Proposition 1(i). However, Doherty and Schlesinger (2002) variable participation indemnity schedule net of its premium and our net variable participating indemnity schedule become identical when the fully participating contract is sold at a fair price.

In their four-state model where the fully participating policy is sold at a fair price, Doherty and Dionne (1993) show the separability of decisions on idiosyncratic risk and on systemic risk; the purchase of full insurance of idiosyncratic risk is independent of the cost of insurance against systemic risk and, therefore, the purchase of partial insurance of systemic risk. Such a result is reconsidered in our multiple state model. From Proposition 1(i), the *form* of the optimal fully participating contract turns out to be independent of the non-participating contract, even if the former is sold at an unfair price. The separability result thus holds about the *form* of the optimal fully participating policy; it displays full insurance against the individual loss above a deductible. The optimal deductible D_1 will depend on the non-participating contract, except when the variable premium P is actuarially fair because, from Proposition 2(i), the optimal level of deductibility is zero, i.e., $D_1 = 0$. This particular case corresponds to the result of Doherty and Dionne (1993). On the contrary, the form of the optimal non-participating contract and its optimal deductible clearly depend on the fully participating contract through the variable premium, as shown in Proposition 1(i).

4. The Financing of Catastrophe Risk with ART Solutions

4.1. Alternative Risk Transfer Solutions

Companies have been using non-traditional forms of risk management for many years, usually called alternative risk transfer (ART) solutions. These innovations aim at increasing the efficiency of the risk transfer, broadening the spectrum of insurable risks and using the capital markets for additional financial capacity. Among the important forms of ART solutions, we focus on two of them; the captives and the insurance derivatives. They are based on the mutuality principle and the transfer principle, respectively.

ART solutions first described various forms of self-insurance, including captives or risk-retention groups. The captive market is of particular interest because it rests on the application of the mutuality principle. A captive is an insurance or reinsurance vehicle. It belongs to a company or a group of companies but it is not active as an insurance firm. It is called single parent captive and collective captive, respectively.[10] Its purpose is to insure risk exposures from its parent company. The role of a captive is twofold. First, it provides a vehicle of self-insurance for high frequency/low severity risks. These "mutual-like" companies allow the firms to monitor managerial performance at lower cost than traditional insurance companies (Doherty and Dionne 1993). Second, it is used as a financing instrument for specific low frequency/high severity risks for which coverage is limited or unavailable on the reinsurance market. The development of group captives, especially in the U.S., is usually viewed as the direct consequence of the liability crises in some economic sectors, such as oils or chemicals (see, for example, Doherty, Kleindorfer and Kunreuther 1990). The coverage solutions offered by the traditional insurance and reinsurance markets to deal with emergent collective risks were considered as too limited or too expensive. Therefore, the lack of capacity and the cost of insurance forced companies facing the same type of risks to create alternative risk management solutions through the development of captives. According to the 1999 issue of Captive Insurance Company Reports, there were about 4,000 captives operating worldwide at year-end 1998, generating a premium volume of approximately 6% of the global commercial insurance market (see Swiss Re (1999) for a detailed description of the importance of captives). More recently, the terrorist attacks in New-York and Washington have induced insurance companies and airline companies to create group captives in order to manage terrorist risks. Likewise, the explosion in AZF fertilizer factory in Toulouse (France) in September 21st 2001 has induced firms exposed to potential industrial catastrophes to create a specific pool to cope with such catastrophic risks.

Following Hurricane Andrew and the Northridge earthquake in the 1990s, property catastrophe reinsurance was in short supply and, as a consequence, premium rates increased sharply. In view of these capacity limits and related limits of insurability, the idea arose of making available additional capacity for catastrophe risks outside the insurance market. Some insurers thus began developing a new class of financial instruments that transfer insurance risk to capital markets. The first attempt to use financial market instruments for managing insurance risks was made by the Chicago Board of Trade (CBOT) which has been trading futures on catastrophe loss indices and related options since December 1992. The PCS options contracts traded on the CBOT are based on various catastrophe loss indices calculated and published on a daily basis using estimates of insured loss. Recent years have witnessed a growing interest in weather-based derivatives as instruments for sharing risk due to weather phenomena.

[10] Other types of captive structures are association captives, risk retention groups and rent a captive arrangements.

Since 1997, market participants in the electricity and natural gas sectors have used temperature-based derivatives to offset their exposure to extreme temperature. Over-the-counter derivatives are based on temperature index such as cumulative heating degree days (HDD) or cooling degree days (CDD) for a given location over a specified period of time. On September 1999, the Chicago Mercantile Exchange began trading standardized monthly cumulative HDD and CDD futures and options contracts for several U.S. cities.[11] Such contracts are being developed in Europe, and especially in France. The European board of trade Euronext plans to launch weather derivatives based on one national and five regional indices in 2003.[12] They will complete the six weather-based futures launched on the British board of trade Liffe in December 2001.[13] The efficient index for a company in its hedging strategy is the result of a tradeoff between basis risk (because these indices are not perfectly correlated with its loss), moral hazard, transaction cost and counterparty risk.

4.2. Optimal hedging strategy

We examine how the two ART mechanisms (captives and index-based derivatives) can be combined to manage efficiently the firm's loss exposure. The risk-averse firm belongs to a collective captive providing fully participating contract $[I(x,e), P(x)]$. It can also buy insurance derivatives based on the systemic component $\tilde{x}$.[14] The indemnity payoff and the premium of this financial contract are denoted $K(.)$ and Q, respectively. In addition, we assume that financial contracts are sold at a price that is proportional to their expected indemnity payoff: $Q = (1+d_2)EK(\tilde{x})$, with $d_2 \geq 0$. The problem considered in this section is slightly different from the previous one because the non-participating policy offers coverage only on the $\tilde{x}$ systemic risk. This thus creates a source of market incompleteness. Following the same line of reasoning as in Proposition 1, one can easily show that the optimal hedging strategy is to insure the idiosyncratic component through the fully participating policy offered by the collective captive and the systemic component through the insurance derivative contract offered by the financial markets. Formally, under multiplicative idiosyncratic and systemic risks, the optimal fully participating policy satisfies

(9) $$I^*(x,e) = \max\left(xe - D_1, 0\right) \text{ with } P(x) = c\left[E\max\left(x\tilde{e} - D_1, 0\right)\right],$$

where $D_1 = 0$ if $c'(.) \equiv 1$ and $D_1 > 0$ otherwise, and the optimal index-based contract is

(10) $$K^*(x) = \max(P(x) - D_2, 0) \text{ with } Q = (1+d_2)E\max(P(\tilde{x}) - D_2, 0),$$

where $D_2 = 0$ if $d_2 = 0$ and $D_2 > 0$ otherwise. The indemnity schedule and the premium of the optimal variable participating policy are $V^*(x,e) = I^*(x,e) + K^*(x)$ and

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[11] Atlanta, Chicago, Cincinnati, Dallas, Des Moines, Las Vegas, New York, Philadelphia, Portland and Tucson. See www.cme.com for more details.

[12] Regional indices will be based on the daily average temperature in five French cities: Ajaccio, Bordeaux, Lyon, Paris-Orly and Strasbourg. See www.euronext.com for more details.

[13] See www.liffe.com for more details.

[14] We assume away problems associated with basis risk or counterparty risk.

$T(x) = P(x) + Q$, respectively. For all x such that $K^*(x) > 0$, the first derivative if the index-based contract is

$$(11) \qquad K'^*(x) = P'(x) = c'(B) \int_{D_1/x} edF(e) \geq 0,$$

where $F(.)$ is the cumulative distribution function of the $\tilde{e}$ risk and $B = E\max(x\tilde{e} - D_1, 0)$. Its second derivative is

$$(12) \qquad K''^*(x) = P''(x) = c''(B)\left[\int_{D_1/x} edF(e)\right]^2 + c'(B)\frac{D_1^2}{x^3} f(D_1/x).$$

From equation (11), the optimal financial contract is a non-decreasing function of the index. From equation (12), it is convex if the cost function of the fully participating policy is linear or convex, $c'' \geq 0$. Its curvature is ambiguous if the cost function is concave. This cost function thus plays a central role in the curvature of the optimal insurance derivative contract.

Suppose first that the premium of the fully participating policy is unfair and non-concave, i.e., $c'(.) > 1$ and $c''(.) \geq 0$. The optimal index-based contract is an increasing and convex function of the x component. Because real-world financial markets offer piecewise linear hedging instruments, like options, they preclude from replicating the first-best hedging strategy. This thus creates a new source of incompleteness. It is noteworthy that even if this contract is sold at a fair price, implying $D_2 = 0$, options contracts turn out to be useful instruments in order to replicate as close as possible the convex curvature of the first-best solution K^*, as long as the premium of the fully participating contract is unfair. Consider now the case where the fully participating policy is sold at a fair price. This entails that $D_1 = 0$ and, from equation (9), $I^*(x,e) = xe$ with $P(x) = (E\tilde{e})x$. The optimal index-based contract thus satisfies

$$(13) \qquad K^*(x) = (E\tilde{e})\max(x - D_2/E\tilde{e}, 0).$$

The first best solution is replicated by purchasing $E\tilde{e}$ call options at a strike price $D_2/E\tilde{e}$. If this index-based contract is sold at a fair price, $d_2 = 0$, then $K^*(x) - Q = (E\tilde{e})(x - E\tilde{x})$; the optimal strategy is to sell a quantity $E\tilde{e}$ of unbiased futures contracts.

4.3. Participating policy as a market enhancing instrument

Real-world markets are typically incomplete in that they offer separate contracts for separate loss exposures. Let $[S_1(e), R_1]$ and $[S_2(x), R_2]$ denote separate hedging contracts with fixed premium against the $\tilde{e}$ risk and the $\tilde{x}$ risk, respectively. We assume that the premiums are proportional to the expected indemnity and that the cost of insurance/hedging is identical for insurance policies dealing with the idiosyncratic risk and for hedging contracts coping with the systemic risk; $P(x) = (1 + d_1)EI(x, \tilde{e})$,

$R_1 = (1+d_1)ES_1(\tilde{e})$, $R_2 = (1+d_2)ES_2(\tilde{e})$ and $Q = (1+d_2)EK(\tilde{x})$. It is noteworthy that the contract $[S_1(e), R_1]$ is a special case of the fully participating policy $[I(x,e), P(x)]$ in which the indemnity does not depend on the realized systemic loss. This implies that any combination of separate non-participating contracts $[S_1(e), R_1]$ and $[S_2(x), R_2]$ is weakly dominated by the variable participating policy $[V(x,e), T(x)]$. This weak dominance is illustrated in two special cases.

Suppose first that the idiosyncratic and the systemic components of the loss exposure are additive; $l(x,e) = x + e$. There exist $D_1 \geq 0$ and $D_2 \geq 0$ such that $I^*(x,e) = \max(x + e - D_1, 0)$ and $K^*(x) = \max(P(x) - D_2, 0)$ with $P(x) = (1+d_2)EI^*(x,\tilde{e})$. From Gollier and Schlesinger (1995), there exist $d_1 \geq 0$ and $d_2 \geq 0$ such that $S_1^*(e) = \max(e - d_1, 0)$ and $S_2^*(x) = \max(x - d_2, 0)$. If insurance contracts covering the $\tilde{e}$ risk are sold at a fair price, i.e., $d_1 = 0$, this implies that $D_1 = d_1 = 0$. This gives

(14) $$I^*(x,e) - P(x) = (x + e) - (x + E\tilde{e}) = (e - E\tilde{e}) = S_1^*(e) - R_1.$$

In addition, we have $K^*(x) = \max(x + E\tilde{e} - D_2, 0) = S_2^*(x)$ with $d_2 = D_2 - E\tilde{e}$. The optimal variable participating policy can thus be replicated with separate contracts, as shown by Doherty and Schlesinger (2002). However, the variable participating policy strictly dominates any combination of separate contracts when insurance against the $\tilde{e}$ risk is costly, i.e., $d_1 > 0$.

Consider now the case in which the idiosyncratic and systemic components of the loss exposure are multiplicative; $l(x,e) = xe$. Such restrictions on hedging contracts abound in the real world. For example, firms facing both price and production uncertainty have usually the possibility to cover their quantity risk with an insurance contract and their price risk with a financial product. The optimal form of separate non-participating contracts for multiplicative risks is first examined. With separate contracts against positive random losses $\tilde{z}_1$ and $\tilde{z}_2$, the problem of the firm is

(15) $$\begin{cases} \max\limits_{S_1,S_2,R_1,R_2} Eu\left(w_0 - \tilde{z}_1\tilde{z}_2 + S_1(\tilde{z}_1) - R_1 + S_2(\tilde{z}_2) - R_2\right) \\ \text{subject to} \\ R_i = (1+d_i)ES_i(\tilde{z}_i), \qquad i = 1,2 \\ S_i(z_i) \geq 0 \text{ for all } z_i, \qquad i = 1,2 \end{cases}$$

with $d_i \geq 0$, $i = 1,2$. It can be shown (see the Appendix) that there exists $d_i \geq 0$, $i = 1,2$, such that the optimal hedging contract satisfies, for $i = 1,2$,

(16) $$S_i^*(z_i)\begin{cases} = 0 & \text{for all } z_i \leq d_i \\ > 0 & \text{for all } z_i > d_i \end{cases}$$

and the marginal coverage satisfies for all $z_i > d_i$,

(17) $$S_i'^*(z_i) = E_j \tilde{z}_j + \mathrm{cov}_j\left(\tilde{z}_j, \frac{u''(\tilde{p})}{E_j u''(\tilde{p})}\right) \geq 0 \quad \text{with} \quad j \neq i,$$

where E_j and cov_j are the expectation and the covariance operators, respectively, with respect to $\tilde{z}_j$, $j = 1,2$ and $p = w_0 - z_1 z_2 + S_1(z_1) - R_1 + S_2(z_2) - R_2$. The marginal coverage of the z_i loss is thus higher or lower than the expected loss $E_j \tilde{z}_j$ depending on whether the covariance term is positive or negative. In addition, the optimal level of deductibility is zero, $d_i = 0$, if and only if the contract is sold at a fair price, $d_i = 0$. It is of interest to notice that, contrary to the previous combination of hedging contracts with a fully participating policy, the separation result does not hold any longer; the *form* of the insurance contract against the $\tilde{z}_1 \equiv \tilde{e}$ risk depends on the indemnity schedule against the $\tilde{z}_2 \equiv \tilde{x}$ risk as long as the firm's utility function is not quadratic, i.e., $u''' \neq 0$.

The optimal hedging strategies with separate non-participating contracts and with the variable participating policy are examined when the idiosyncratic and systemic components interact in a multiplicative manner in the loss exposure, insurance policies are sold at an unfair price, $d_1 > 0$ and $d_2 > 0$, and the firm's preferences are represented by a quadratic utility function.[15] The covariance term in (17) is zero and the optimal hedging contracts become

(18) $$S_i^*(z_i) = E\tilde{z}_j \max(z_i - d_i, 0) \quad \text{with} \quad j \neq i,$$

for $i = 1,2$. The optimal hedging strategy thus entails buying $E\tilde{z}_2$ call options at a strike price d_1 and $E\tilde{z}_1$ call options at a strike price d_2. The profit of the insured firm is

(19) $$p^{NP} = w_0 - xe + E\tilde{x} \max(e - d_1, 0) + E\tilde{e} \max(x - d_2, 0) - R_1 - R_2,$$

where $\tilde{z}_1 \equiv \tilde{e}$ and $\tilde{z}_2 \equiv \tilde{x}$. When the fully participating policy is available, the profit of the insured firm is

(20) $$p^{VP} = w_0 - xe + \max(xe - D_1, 0) - P(x) + \max(P(x) - D_2, 0) - Q.$$

The efficient frontiers of these hedging strategies are depicted in Figure 1. The initial wealth position of the firm is shown as *A*. If the $\tilde{x}$ systemic risk can only be hedged or insured, the efficient frontier is the line *A-B*. This line shows how the firm's wealth can be covered with a single contract against the $\tilde{x}$ systemic risk. A reduction in the variance of wealth is associated with a reduction in the expected wealth because the premium is higher than the actuarial value of the contract, $d_2 > 0$. This optimal strategy entails buying $E\tilde{e}$ call options; $H^*(x) = E\tilde{e}\max(x - d_3, 0)$ with $d_3 > 0$. If the deductible is set at zero, $d_3 = 0$, the profit of the firm satisfies $p^S\big|_{d_3=0} = w_0 - xe + xE\tilde{e} - (1 + d_2)E\tilde{e}E\tilde{x}$ and thus its expectation and its variance are, respectively,

[15] The main actors in this market are firms, not individuals, and linear mean variance preferences are well accepted in the corporate finance literature.

(21) $E\tilde{p}^{S}\big|_{d_3=0} = w_0 - (1+d_2)E\tilde{e}E\tilde{x}$ and $\text{var}(\tilde{p}^{S})\big|_{d_3=0} = \text{var}(\tilde{x}(\tilde{e} - E\tilde{e})) > 0$,

where var(.) is the variance operator. This is the coordinates of point *B*. There is still basis risk remaining because the $\tilde{e}$ risk is unhedgeable/uninsurable. If separate non-participating contracts are available for the two independent sources of risk, the efficient frontier is the line *A*-*C*. This line shows the potential choices available to the purchaser of the hedge with separate contracts. Because the hedging strategy with only one hedging contract against the $\tilde{x}$ risk is a special case of the hedging strategy with two separate non-participating polices, the latter dominates the former. When the deductibles are set at zero, $d_1 = d_2 = 0$, the profit of the firm expressed in (19) becomes

(22) $p^{NP}\big|_{d_1=d_2=0} = w_0 - xe + E\tilde{x}e + E\tilde{e}x - (2+d_1+d_2)E\tilde{x}E\tilde{e}$,

with

(23) $E\tilde{p}^{NP}\big|_{d_1=0,d_2=0} = w_0 - (1+d_1+d_2)E\tilde{e}E\tilde{x}$ and $\text{var}(\tilde{p}^{NP})\big|_{d_1=0,d_2=0} = \text{var}((\tilde{x} - E\tilde{x})(\tilde{e} - E\tilde{e})) > 0$.

This is the coordinates of point *C*. Incomplete markets caused by separate additive contracts to manage multiplicative random losses prevent the firm from having a non-random final wealth. There is basis risk remaining due to this incompleteness. From equations (21) and (23), we have $E\tilde{p}^{NP}\big|_{d_1=0,d_2=0} < E\tilde{p}^{S}\big|_{d_1=0}$ if, and only if, $d_2 > 0$. In addition the basis risk is lower when two separate contracts are available because $\text{var}(\tilde{p}^{NP})\big|_{d_1=0,d_2=0} < \text{var}(\tilde{p}^{S})\big|_{d_1=0}$. When the variable participating policy is at the firm's disposal, the efficient frontier is the line *A*-*D*. It shows the potential choices available to the purchaser of the hedge with the variable participating policy. With full coverage under each policy, $D_1 = D_2 = 0$, the final wealth of the firm expressed in equation (20) becomes

(24) $p^{VP}\big|_{D_1=0,D_2=0} = w_0 - (1+d_1)(1+d_2)E\tilde{x}E\tilde{e}$,

with

(25) $E\tilde{p}^{VP}\big|_{D_1=0,D_2=0} = w_0 - (1+d_1)E\tilde{e}(1+d_2)E\tilde{x}$ and $\text{var}(\tilde{p}^{VP})\big|_{D_1=0,D_2=0} = 0$.

This is the coordinates of point *D*. The final wealth of the firm becomes non-random, i.e., the firm is fully covered against both components of the risky loss. Observe that, from equations (23) and (25), $E\tilde{p}^{VP}\big|_{D_1=0,D_2=0} < E\tilde{p}^{NP}\big|_{d_1=0,d_2=0}$ if $d_1 > 0$ and $d_2 > 0$, and $E\tilde{p}^{VP}\big|_{D_1=0,D_2=0} = E\tilde{p}^{NP}\big|_{d_1=0,d_2=0}$ otherwise. The hedging strategy with the variable participating policy clearly dominates the hedging strategy with separate non-participating contracts. Therefore, group captives dealing with idiosyncratic risk (through fully participating policy) increase market efficiency in the management of systemic risk. This market enhancement is twofold. First, reducing risk (in terms of variance) is less costly (in term of expected wealth) with the variable participating policy than with separate non-participating contracts. Second, the former strategy offers the (infinitely risk-averse) firm the opportunity to be fully covered against random loss, contrary to the latter strategy which contains unhedgeable basis risk.

[INSERT FIGURE 1 HERE]

The superiority of the variable participating contract over the combination of separate contracts as illustrated in Figure 1 is closely related to the costs of hedging which has been assumed to be identical for the non-participating contract and the fully participating policy on the $\tilde{e}$ risk, and for the hedging contracts on the $\tilde{x}$ risk. Suppose now that the price of hedging the systemic component is fair, $d_2 = 0$, while the hedging cost of the fully participating policy, d_1, is higher than that of the non-participating contract on the idiosyncratic risk, d_1'. The efficient frontiers of hedging strategies are depicted in Figure 2. Because the systemic risk can be hedged at a fair price, there is no loss in expected wealth when this contract is purchased and, therefore, the efficient frontier of the single contract is the horizontal line *A*-*B*. Position *B* shows full coverage on the systemic risk. The remaining risk is due to the idiosyncratic loss. This position can also be reached with separate non-participating contracts and with the variable participating policy. Lines *B*-*C* and *B*-*D* show additional coverage on the idiosyncratic risk with the non-participating contract and the fully participating policy, respectively. There is basis risk remaining in position *C* because the separate non-participating contract do not allow the firm to be fully hedged against its loss exposure, while position *D* shows full coverage of the loss exposure. However, line *B*-*C* is above line *B*-*D* because hedging the idiosyncratic risk is less costly under the non-participating contract than under the fully participating policy, $d_1 > d_1'$. Therefore, the variable participating contract does not dominate the hedging strategy with separate contracts. The choice between these two strategies rests on risk preferences: highly risk-averse firms would prefer the variable participating policy (see indifference curve IC_h), while moderately risk-averse firms would prefer separate contracts (see indifference curve IC_m). The choice is thus based on a tradeoff between basis risk and transaction costs.

[INSERT FIGURE 2 HERE]

5. Conclusion

This paper presents a normative model to investigate the role of fully participating and non-participating insurance policies in the management of catastrophic risk. The design of these two optimal contracts is derived from a decomposition of the individual loss into idiosyncratic and systemic components. When the cost of insurance of the fully participating contract is lower than that of the non-participating policy, the optimal risk-sharing arrangement is shown to be as follows. Under the fully participating policy, the indemnity schedule displays full insurance above a deductible on the individual loss and the premium depends on the realized systemic loss. The *form* of this contract is independent of the non-participating contract. Under the non-participating policy, the indemnity schedule displays full insurance above a deductible on the variable premium of the fully participating policy. The payoff thus depends only on the systemic component of the individual loss. Hence, the systemic risk is first filtered through the participating policy and then it is transferred through an insurance contract providing partial a coverage on the variable premium. These two separate optimal contracts are combined to design an optimal variable participating insurance policy in which the degree of participation is endogenous.

This risk-sharing problem is reconsidered to investigate the financing of catastrophe risk with two ART solutions; group captives and index-based contracts. We show that the first-best hedging contract against the systemic risk cannot be replicated with existing piecewise-linear instruments (e.g., options) when the premium of the fully participating policy is unfair. This thus creates a source of market incompleteness. Fully participating insurance contracts provided by group captives are shown to be market enhancing. This is illustrated from the analysis of the dominance relationship using the efficient frontier of the optimal variable participating policy and the efficient frontier of the hedging strategy based on two separate non-participating contracts in the mean-variance framework.

Our model is also useful to investigate the risk management of hybrid covers. Insurance companies are traditionally exposed to two types of risk: insurance risk from policyholders' claims and financial risks on assets. Hybrid covers are innovative hedging instruments which provide coverage on joint insurance and financial market risks, e.g., stop loss insurance cover where the retention varies in proportion to an equity index. While the insurance risk can be covered via traditional techniques, the financial risk must be transferred via financial hedging tools. Optimal risk management of hybrid covers is thus based on an appropriate decomposition of the hybrid risk into a pure finance risk and an independent hybrid residual risk in order to apply the two fundamental principles in risk allocation. The insurance risk can be viewed as an unhedgeable basis risk from the financial investor's viewpoint. Our model shows how such an hybrid risk can be efficiently covered. The insurance risk, i.e., the idiosyncratic component, is first (partially) covered by a pool of insurers and the financial risk, i.e., the systemic component, is (partially) hedged on financial markets.

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Appendix

Proof of Proposition 1

The design of the optimal non-participating policy is first derived when the premium Q is given. Problem (6) can be solved by using Kuhn-Tucker conditions for $J(x,e)$ for all (x,e) because its first derivatives appear neither in the objective function nor in the constraints. The first-order condition is

(A1)
$$u'(w_0 - l(x,e) + I(x,e) - P(x) + J(x,e) - Q) + l_2(x,e) - m_2 d'(EJ(\tilde{x},\tilde{e})) = 0$$

for all (x,e), where m_2 and l_2 are the Lagrangian multipliers associated respectively to constraints (3) and (2), with

(A2) $$l_2(x,e)\begin{cases} = 0 \text{ if } J(x,e) > 0 \\ \geq 0 \text{ otherwise.} \end{cases}$$

Condition (A1) can be rewritten as

(A3) $$u'(w_0 - l(x,e) + I(x,e) - P(x) + J(x,e) - Q) = m_2 d'(EJ(\tilde{x},\tilde{e}))$$

for all (x,e) such that $J(x,e) > 0$. This implies that the marginal utility of the firm is constant in every state of the world where the indemnity J is paid:

(A4) $$I(x,e) + J(x,e) = l(x,e) + P(x) \text{ for all } (x,e) : J(x,e) > 0 .$$

Consider now the design of the optimal fully participating policy. The first-order condition is

(A5)
$$u'(w_0 - l(x,e) + I(x,e) - P(x) + J(x,e) - Q) + l_1(x,e) - m_1(x) c'(EI(x,\tilde{e})) = 0$$

for all (x,e) ,where $m_1(.)$ and l_1 are the Lagrangian multipliers associated respectively to constraints (5) and (4), with

(A6) $$l_1(x,e)\begin{cases} = 0 \text{ if } I(x,e) > 0 \\ \geq 0 \text{ otherwise.} \end{cases}$$

Condition (A5) becomes

(A7) $$u'(w_0 - l(x,e) + I(x,e) - P(x) + J(x,e) - Q) = m_1(x) c'(EI(x,\tilde{e}))$$

for all (x,e) such that $I(x,e) > 0$. Consequently, for every level of systemic risk x, the marginal utility of the firm must be constant with respect to the idiosyncratic variable:

(A8) $$I(x,e) + J(x,e) = l(x,e) + P(x) \text{ for all } e : I(x,e) > 0 .$$

First, suppose that the cost of coverage is higher under the non-participating policy than under the fully participating contract: $c(s) < d(s)$ for all s. The firm will choose to be insured against the $\tilde{e}$ risk through the less expensive contract, i.e., the fully participating policy and, consequently, $J_e^* = 0$. Then it will select a coverage against the $\tilde{x}$ risk through the non-participating contract. This leads to part (i) of Proposition 1.

Second, suppose that $c(s) > d(s)$ for all s. It is less expensive to purchase the non-participating policy than the fully participating contract. Full insurance against the individual loss above a straight deductible is thus optimal. This leads to part (ii) of Proposition 1.

Proof of Proposition 2

Consider the optimal level of deductibility under the non-participating insurance policy. The optimization of the objective function in (6) with respect to Q yields the first-order condition

(A9) $$m_2 = Eu'(p(\tilde{x},\tilde{e}))$$

where $p(x,e) = w_0 - l(x,e) + I(x,e) - P(x) + J(x,e) - Q$. The first-order condition (A1) becomes

(A10) $$l_2(x,e) = -u'(p(x,e)) + Eu'(p(\tilde{x},\tilde{e}))d'(EJ(\tilde{x},\tilde{e})).$$

Taking the expectation with respect to $(\tilde{x},\tilde{e})$ yields

(A11) $$E\,l_2(\tilde{x},\tilde{e}) = Eu'(p(\tilde{x},\tilde{e}))[d'(EJ(\tilde{x},\tilde{e}))-1].$$

If $d'(s) = 1$ for all s, then $E\,l_2(\tilde{x},\tilde{e}) = 0$. From the definition of l_2 in (A2), this implies that $l_2(x,e) = 0$ for all (x,e) almost surely and, consequently, from Proposition 1, $D_2 = 0$. If $d'(s) > 1$ for all s, then $E\,l_2(\tilde{x},\tilde{e}) > 0$. From the definition of l_2 in (A2), this implies that $l_2(x,e) > 0$ for some (x,e) with a positive probability and, consequently, from Proposition 1, $D_2 > 0$. This leads to part (i) of Proposition 2.

Consider now the optimal level of deductibility under the participating insurance policy. The above procedure can be shown to lead to an ambiguous result because of the presence of the variable premium. An alternative way is to consider the following maximization problem:

(A11) $$\underset{D_1}{Max}\, Eu\left(w_0 - l(\tilde{x},\tilde{e}) + \max\left(l(\tilde{x},\tilde{e}) - D_1, 0\right) - P(\tilde{x}) + J(\tilde{x},\tilde{e}) - Q\right).$$

It is easy to show that the first-order condition of (A11) evaluated at $D_1 = 0$ equal zero if $c'(s) = 1$ for all s, and it is positive if $c'(s) > 1$ for all s. This leads to part (i) of Proposition 2. Notice that this result is derived assuming the second-order condition holds.

Optimal separate non-participating insurance contracts

When the premium R_i is fixed, using Kuhn-Tucker conditions for $S_i(z_i)$ for all z_i yields the first-order condition of the maximization problem (15):

(A12) $$E_j u'\left(w_0 - z_i\tilde{z}_j + S_i(z_i) - R_i + S_j(\tilde{z}_j) - R_j\right) + l_i(z_i) - (1+d_i)m_i = 0 \quad \text{for all } z_i,$$

where m_i and l_i are the Lagrangian multipliers associated respectively to premium constraint and the non-negativity constraint, with

(A13) $$l_i(z_i)\begin{cases} = 0 \text{ if } S_i(z_i) > 0 \\ \geq 0 \text{ otherwise.} \end{cases}$$

For all z_i such that $S_i(z_i) = 0$, we have

(A14) $$V_i(z_i) = E_j u'\left(w_0 - z_i\tilde{z}_j - R_i + S_j(\tilde{z}_j) - R_j\right) - (1+d_i)m_i \leq 0 .$$

V_i decreases with z_i because $u'' < 0$ and $z_j \geq 0$. This implies that there exists a deductible d_i such that the optimal insurance contract satisfies equation (16).

For all z_i such that $S_i(z_i) > 0$, differentiating (A12) with respect to z_i gives

(A15) $$S_i^{*}{}'(z_i) = \frac{E_j\left(\tilde{z}_j u''(\tilde{p})\right)}{E_j u''(\tilde{p})},$$

where $\tilde{p} = w_0 - z_i\tilde{z}_j + S_i(z_i) - R_i + S_j(\tilde{z}_j) - R_j$. The marginal indemnity function is positive. Decomposing the numerator of (A15) leads to equation (17).

Optimizing problem (15) with respect to R_i yields

(A16) $$m_i = E_i E_j u'\left(w_0 - \tilde{z}_i\tilde{z}_j + S_i(\tilde{z}_i) - R_i + S_j(\tilde{z}_j) - R_j\right).$$

Introduction (A16) in (A13) and taking the expectation with respect to $\tilde{z}_i$ gives

(A17) $$E\, l_i(\tilde{z}_i) = d_i E_i E_j u'\left(w_0 - \tilde{z}_i\tilde{z}_j + S_i(\tilde{z}_i) - R_i + S_j(\tilde{z}_j) - R_j\right).$$

This implies that the optimal deductible d_i is zero if, and only if, the loading factor d_i is zero.

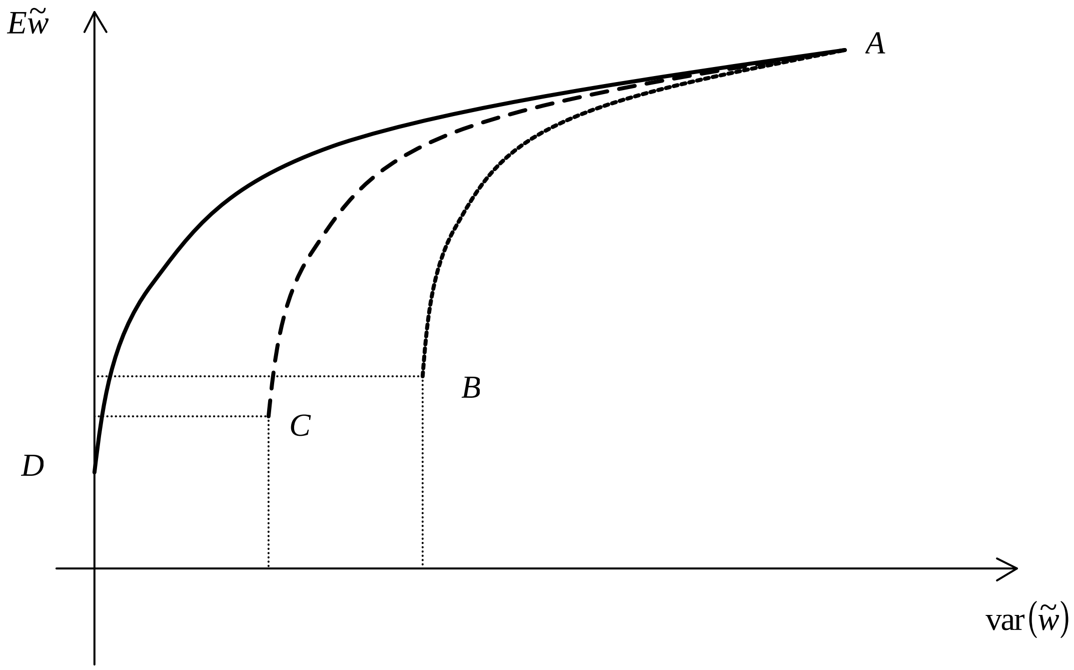

Figure 1. Efficient frontiers under a single non-participating contract on the systemic risk (line *A*-*B*), separate non-participating contracts (line *A*-*C*), and variable participating contract (line *A*-*D*); $d_1 > 0, d_2 > 0$.

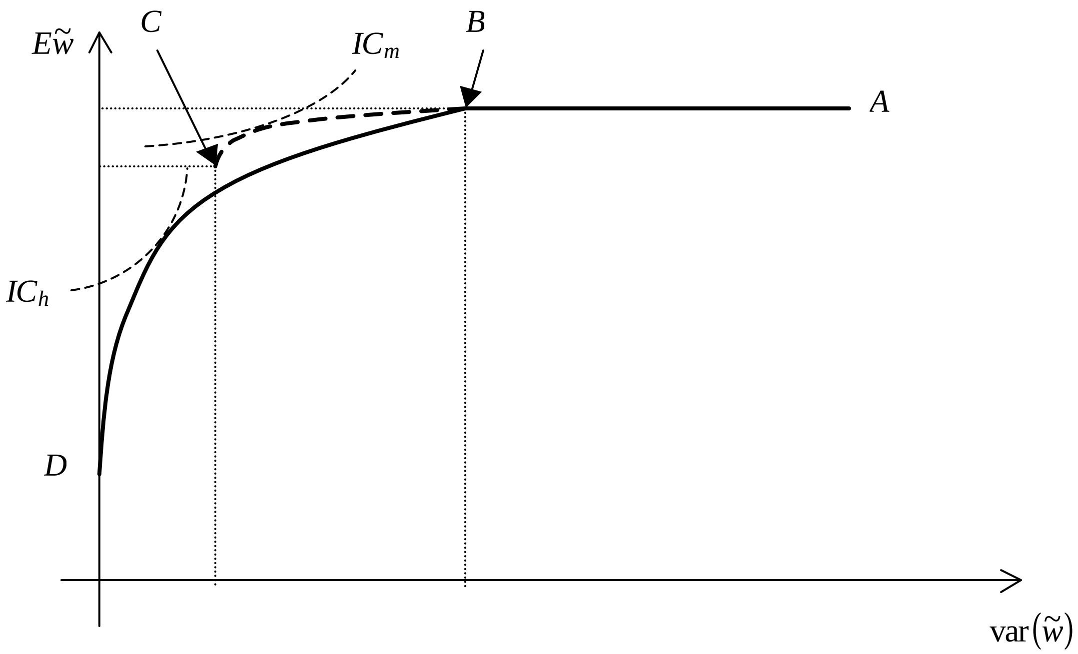

Figure 2. Efficient frontiers under a single contract on the systemic risk (line *A*-*B*), separate non-participating contracts (line *A*-*C*), variable participating contract (line *A*-*D*); $d_1 > d_1', d_2 = 0$.