How Much is a QALY Worth?
Admissible Utility Functions for Health and Wealth

James K. Hammitt
Harvard University

Department of Health Policy and Management
and Center for Risk Analysis
School of Public Health
718 Huntington Ave.
Boston, MA 02115 USA

tel: 617/432-4030, fax: 617/432-0190
e-mail: jkh@hsph.harvard.edu

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Abstract

Utility functions for health, longevity, and lifetime wealth are derived under the assumption that the utility of health and longevity, conditional on lifetime wealth, can be represented by risk-adjusted quality-adjusted life years (QALYs). Some commonly used utility functions (e.g., expected discounted period utility) are inconsistent with these assumptions except in special cases. The admissible utility functions imply that an individual's willingness to pay (WTP) per QALY is not constant, but depends on health, longevity, wealth, and risk posture with respect to longevity. Under reasonable additional assumptions, WTP per QALY increases with lifetime wealth, but the effects of health and longevity are ambiguous.

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1. Introduction

Utility depends on health, longevity, and wealth or income which can be used for other consumption. The effects of these factors are of interest both for modeling individual behavior and for evaluating policies that affect health and longevity. Parameters of particular interest include individuals' marginal rates of substitution between health, longevity, and consumption.

In cost-effectiveness analysis (CEA), it is typically assumed that preferences for health and longevity can be represented by the associated number of quality-adjusted life years (QALYs), and policies are evaluated in terms of the cost incurred per QALY gained. Determining whether the cost per QALY is acceptable or excessive requires comparison with some measure of willingness to pay (WTP) per QALY. The question of whether CEA is consistent with economic welfare theory and can be interpreted as a form of benefit-cost analysis (BCA) depends in part on whether individual WTP per QALY can be assumed to be constant across individuals and for different changes in QALYs (Johannesson, 1995; Garber and Phelps, 1997).[1]

Estimates of WTP per QALY are also useful for combining results from the QALY and WTP literatures in order to value changes in health. For example, Tolley et al. (1994) and Cutler and Richardson (1997) measure the value of health by multiplying changes in QALYs by an estimate of the value per statistical life year (VSLY) (Moore and Viscusi, 1988). Johnson et al. (1997) estimate a less restrictive transfer function to predict WTP to avoid morbidity using a QALY-based measure of health quality and duration of the illness and Liu et al. (2000) use these factors to describe household WTP to protect a mother or child from suffering a cold.

In previous work, the effects of health, longevity, and consumption on utility have been represented by a variety of utility functions. Grossman (1972) assumed a very general function,

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[1] Johannesson (1995: 485) writes "the difference between cost-benefit analysis and cost-effectiveness analysis is that in cost-effectiveness analysis the willingness to pay per QALY gained is assumed to be the same for all individuals under all circumstances and for all sizes of the change in QALYs."

$$U = u(h_1, h_2, ..., h_T, c_1, c_2, ..., c_T), \tag{1.1}$$

where h_t is the flow of health services and c_t is other consumption in period t. Subsequent authors have imposed more structure, often assuming that lifetime utility is the expected discounted sum of period utilities,

$$U = \sum_{t=0}^{T} \delta^t s_t u(h_t, c_t), \tag{1.2}$$

where δ is a discount factor, s_t is the probability of surviving to age t, and T is the maximum possible age (e.g., Cutler and Richardson, 1997; Meltzer, 1997). The within-period utility function in equation (1.2) is often assumed to be a multiplicative function of health and consumption,

$$u(h_t, c_t) = q(h_t)\, v(c_t), \tag{1.3}$$

where q(•) and v(•) are monotonically increasing functions (e.g., Garber and Phelps, 1997). Bleichrodt and Quiggin (1999) provide an axiomatic basis for the representation in equations (1.2–1.3) under both expected utility and rank-dependent expected utility theories. Alternative representations include the monetary-loss-equivalent model, in which the effect of impaired health is equivalent to the effect of reduced consumption,

$$u(h, c) = u[h^*, c - m(h)], \tag{1.4}$$

where h* represents perfect health and m(h) is a monetary value of health h (Evans and Viscusi, 1990).

The literature on valuing mortality risk typically uses a health-state-dependent specification,

$$u(h, c) = u_h(c), \tag{1.5}$$

where the health state is restricted to two values, alive and dead (e.g., Drèze, 1962; Jones-Lee, 1974; Weinstein et al., 1980; Pratt and Zeckhauser, 1996). Equation (1.5) is also used to model preferences for different health states while living (Evans and Viscusi, 1990; Sloan et al., 1998). The health-state model (1.5) is equivalent to u(h, c) as used for within-period utility in equation (1.2).

In this paper, I characterize the specifications of the lifetime utility function for health, longevity, and wealth that are consistent with the assumption that preferences over health and longevity can be represented by QALYs. I then examine the properties of these admissible specifications, including their implications regarding willingness to pay to increase health, longevity, and probability of survival.

The paper is organized as follows. Section 2 describes the assumptions under which QALYs describe preferences for health and longevity, identifies an additional assumption that is implicit in the use of QALYs, and derives the utility functions for health, longevity, and wealth that are consistent with these assumptions. Section 3 characterizes the implications of the admissible utility functions for the marginal utility of consumption, the risk postures with respect to longevity and wealth, and the dependence of these properties on health, longevity, and wealth. WTP per QALY and its dependence on health, longevity, and wealth are examined in Section 4, and WTP to decrease current-period mortality risk (the "value per statistical life") is examined in Section 5. Section 6 concludes.

2. Utility for Health, Longevity, and Wealth

Let lifetime utility u(h, L, w) depend on health h, longevity L, and wealth w.[2] The value of h may be assumed constant over the lifetime, or may represent an average or other representative lifetime value. This approach contrasts with previous work that assumes lifetime utility can be represented as a discounted sum of period utilities (equation (1.2)), which requires assumptions about intertemporal separability, additivity, and the form of the discounting function (e.g., Shepard and Zeckhauser, 1984; Ng, 1992; Bleichrodt and Quiggin, 1999; Harvey, 1994).

Assume that preferences over health and longevity are such that, holding wealth constant at any value w', the individual prefers more QALYs to fewer. Although the standard form of QALY ubiquitous in the literature assumes risk neutrality over longevity, Pliskin et al. (1980) propose a more general form of QALY which includes other risk postures. Pliskin et al. assume that preferences satisfy mutual utility independence (preferences for lotteries on health or on longevity, holding the value of the other attribute fixed, are independent of the value at which that attribute is fixed) and

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[2] Lifetime wealth includes labor and other income that is available for spending on consumption goods and for leaving as a bequest.

constant proportional tradeoff of longevity for health.[3] Under these assumptions, the conditional utility function for health and longevity holding wealth constant at some value w' can be represented as

$$u(h, L \mid w') = [q(h)\, L]^r \qquad r > 0 \qquad (2.1a)$$

$$= \log[q(h)\, L] \qquad r = 0 \qquad (2.1b)$$

$$= -\,[q(h)\, L]^r \qquad r < 0 \qquad (2.1c)$$

where q(h) is the "health-related quality of life" (HRQL[4]) associated with health h.

The utility function (2.1a–2.1c) exhibits constant relative risk posture with measure 1 – r. If r = 1, the individual is risk neutral over longevity; if r < 1, he is risk averse; and if r > 1, he is risk seeking. In contrast to the case of wealth, where risk-seeking preferences are considered unusual if not normatively implausible, risk-seeking preferences with respect to longevity appear to be normatively acceptable.[5] In addition, there is descriptive evidence of all three types of risk preference. Pliskin et al. (1980) surveyed ten health-utility experts and report that four expressed risk-seeking preferences, four expressed risk-neutral preferences, and two expressed risk-averse preferences with respect to longevity. In a general-population survey, Corso and Hammitt (2001) asked respondents to choose the preferred lottery from each of five pairs of lotteries on longevity. Few respondents made all five choices consistent with any global risk posture, but for about half the respondents, at least four responses were consistent with a global risk posture. Of the full sample, 16 percent gave at least four responses consistent with

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[3] Bleichrodt et al. (1997) provide an alternative set of conditions for the restricted case of risk neutrality over longevity.

[4] Several terms, including health related quality of life, health status, functional status, and utility weight, are used in the literature to designate a variety of uni- and multidimensional measures of health. I follow the US Public Health Service panel (Gold et al., 1996) and Dolan (2000) in using the term HRQL.

[5] Note that if L represents future longevity from the individual's current age, constant proportional risk posture is dynamically inconsistent. Consider a choice between living 20 years longer and a lottery offering equal chances of living 16 or 25 years longer (where q(h) = 1 for all years). An individual with utility function $[q(h)\, L]^{1/2}$ would choose the lottery. But 16 years later, he would reverse his choice, since he would prefer living 4 years longer to the lottery offering equal chances of living 0 and 9 more years. If L is total life span, there is no inconsistency.

risk proneness, 0.5 percent gave responses consistent with risk neutrality, and 27 percent gave responses consistent with risk aversion.

Note that the utility function (2.1a–2.1c) is inconsistent with the discounted additive utility function (1.2), except in the case of risk neutrality (r = 1) and no discounting (δ = 1). To see this, consider a continuous time version of equation (1.2), and assume for simplicity perfect health and no mortality risk until the individual dies at age T. In this case,

$$U = \int_0^T e^{-\rho t} dt = \frac{1}{\rho}\left(1 - e^{-\rho T}\right), \tag{2.2}$$

where ρ > 0 is the individual's discount rate. Clearly equation (2.2) is not a positive affine transformation of the utility implied by equations (2.1a–2.1c), i.e., T^r (r > 0), log(T), and $-T^r$ (r < 0). Johannesson et al. (1994) describe how QALYs could be consistent with discounted utility if the Pliskin et al. (1980) axioms were modified to replace life years with discounted life years (using the individual's discount rate).

The literature on QALYs is virtually silent on the extent to which HRQL depends on wealth, income, or consumption. In practice, HRQL is elicited with no attention to income or wealth, and so it is assumed (at least implicitly) to be independent of wealth. This assumption will be described as "HRQL invariance." It is analogous to Bleichrodt's and Quiggin's (1999) assumption of "standard gamble invariance," but I adopt the more general term to allow for the use of risk-adjusted QALYs (equations (2.1a–2.1c)). The function $[q(h)]^r$ is a von Neumann-Morgenstern utility function for health and is estimated by the standard gamble. By definition, the individual is risk neutral with respect to health as measured by $[q(h)]^r$. The function q(h), which is estimated by the time-tradeoff method, is a monotonic transformation of the utility of health but is not itself a von Neumann-Morgenstern utility function unless r = 1.[6]

If preferences for health and longevity are consistent with the QALY formulation (2.1a–2.1c) and q(h) satisfies HRQL invariance, then conditional preferences for lotteries

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[6] Pliskin et al. (1980) represent QALYs as $[H_r(q)\ L]^r$, where q is a utility for health and $H_r(q)$ is a positive monotone transformation of q that is nonlinear except when r = 1. The alternative specification used here simplifies notation in the following sections.

on health and longevity holding w fixed do not depend on the level at which w is fixed. Hence the conditional utility functions {u(h, L | w)} must be strategically equivalent (i.e., represent the same preferences over [h, L]), and so these conditional utility functions are related as positive affine transformations (Keeney and Raiffa, 1976). This implies that the utility function for health, longevity, and wealth can be written as

$$u(h, L, w) = u(h, L \mid w)\, a(w) + b(w) \qquad (2.3)$$

where a(w) > 0 (so more QALYs are preferred to fewer).[7] If there is a subsistence wealth level w_s below which the individual prefers death to survival, a(w) may be less than or equal to zero for $w \leq w_s$. Substituting equations (2.1a–2.1c) into equation (2.3) yields:

$$u(h, L, w) = [q(h)\, L]^r\, a(w) + b(w) \qquad r > 0 \qquad (2.4a)$$

$$= \log[q(h)\, L]\, a(w) + b(w) \qquad r = 0 \qquad (2.4b)$$

$$= -[q(h)\, L]^r\, a(w) + b(w) \qquad r < 0. \qquad (2.4c)$$

Equations (2.4a–2.4c) describe the utility functions for health, longevity, and wealth that are admissible under the assumptions that preferences for health and longevity can be represented using risk-adjusted QALYs and that q(h) satisfies HRQL invariance. In the following sections, the restrictions this utility function imposes on the marginal utility of wealth, the risk postures for longevity and wealth, willingness to pay per QALY, and willingness to pay to reduce current mortality risk are examined. For simplicity, attention will be restricted to equation (2.4a). This form includes all three risk postures for longevity (risk averse, risk neutral, and risk seeking). Analogous results can be obtained for equations (2.4b) and (2.4c), both of which require risk aversion for longevity.

3. Properties of the Admissible Utility Functions

In this section, properties of the admissible utility functions are considered, including the marginal utility of wealth, the risk postures with respect to longevity and wealth, and their dependence on other attributes.

Marginal Utility of Wealth

For notational simplicity, let

$$Q = q(h)\ L \tag{3.1}$$

so that equation (2.4a) may be written as

$$u(h, L, w) = Q^r\ a(w) + b(w). \tag{3.2}$$

To evaluate equation (3.2), first consider the case Q = 0. In this case, u(h, L, w) = b(w), so b(w) is the utility function for wealth conditional on death (i.e., the utility of a bequest), which must be equal to the utility of wealth for health states indifferent to death (for which q(h) = 0). In the literature on valuing mortality risk, it is conventionally assumed that b'(w) ≥ 0 (e.g., Jones-Lee, 1974; Weinstein et al., 1980), and I adopt that assumption here.

The marginal utility of wealth is given by

$$\frac{\partial}{\partial w} u(h, L, w) = Q^r a'(w) + b'(w), \tag{3.3}$$

where prime indicates first derivative. I assume that, for health states preferred to death (i.e., Q > 0), the marginal utility of wealth is strictly positive, which implies

$$a'(w) > -\ b'(w)\ /\ Q^r. \tag{3.4}$$

If b'(w) = 0, then a'(w) > 0 and the marginal utility of wealth increases with Q. This implies that the marginal utility of wealth increases with health and with longevity. If b'(w) > 0, then a'(w) may be less than zero so long as its absolute value is not too large (i.e., $a'(w) > - b'(w) / Q_m^r$ where Q_m is the maximum possible value of Q, obtained for HRQL = 1 and a maximal lifespan). If a' < 0, the marginal utility of wealth decreases with health and longevity. The literature on valuing mortality risk assumes that the marginal utility of wealth is greater in the event of survival than in the event of death. Limited empirical evidence (Sloan et al., 1998; Viscusi and Evans, 1990) also supports the notion

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[7] Bleichrodt and Quiggin (1999) derive a similar result for within-period utility $u(h_t, c_t)$ then argue that the utility of a bequest is additively separable from utility of consumption while living, which implies $b(c_t) = 0$ for the within-period utility function (1.3). They do not consider the effects of adding a term representing the utility of a bequest to their utility function (1.2–1.3).

that the marginal utility of wealth increases with health and longevity, which implies a'(w) > 0.

Risk Postures for Longevity and Wealth

The risk posture for longevity is independent of wealth. Depending on the value of r, the individual exhibits constant relative risk aversion (r < 1), risk neutrality (r = 1), or constant relative risk proneness (r > 1) with respect to longevity.

In contrast, the risk posture for wealth depends on health and longevity. The Arrow-Pratt measure of absolute risk aversion (Pratt, 1964) is given by

$$\pi(w) = -\frac{\frac{\partial^2}{\partial w^2} u(q, L, w)}{\frac{\partial}{\partial w} u(q, L, w)} = -\frac{Q^r a''(w) + b''(w)}{Q^r a'(w) + b'(w)}, \tag{3.5}$$

where single and double primes denote first and second derivatives, respectively. Note that the individual's risk posture with respect to wealth is influenced by his risk posture with respect to longevity (the value of r), but is not determined by it. With respect to wealth, the individual may be locally risk averse, risk neutral, or risk seeking (or any combination of these for different wealth levels) regardless of his risk posture for longevity. For the state of life, it is widely assumed that individuals are risk averse with respect to wealth, and there is substantial empirical support for this assumption (e.g., Barksy et al., 1997).

The effect of risk posture with respect to longevity on the risk posture with respect to wealth is complex, and without further assumptions on the first and second derivatives of a(w) and b(w), no general statements can be made. A few special cases are easily solved. If the individual is indifferent with regard to the level of bequest (i.e., b'(w) = 0), then b"(w) = 0 and (for Q > 0)

$$\pi(w) = -\frac{a''(w)}{a'(w)} \tag{3.6}$$

so the risk posture with respect to wealth is independent of health and longevity (except at Q = 0, where π(w) = 0). Similarly, if the degree of absolute risk aversion with respect to wealth is identical for a(w) and b(w), then risk posture is independent of health and longevity and is given by equation (3.6) for all values of Q ≥ 0. If the marginal utility of bequest is positive but the individual is risk neutral regarding his bequest (b'(w) > 0 and b''(w) = 0), then

$$\pi(w) = -\frac{a''(w)}{a'(w) + Q^{-r} b'(w)} \tag{3.7}$$

and so (if a'(w) > 0 and a''(w) < 0) risk aversion increases with health and longevity.

Barsky et al. (1997) report evidence consistent with the prediction that financial risk aversion increases with health and longevity. They find that smokers and heavy drinkers, who have poorer health and shorter life expectancies than others, are less averse to financial risk.

4. Willingness to Pay per QALY

This section examines the implications of the admissible utility functions for willingness to pay (WTP) per QALY and its dependence on health, longevity, and wealth. Let V be the individual's WTP per QALY. V is obtained by totally differentiating equation (3.2) holding utility constant to obtain

$$V = -\frac{dw}{dQ} = \frac{rQ^{r-1} a(w)}{Q^r a'(w) + b'(w)} + \frac{\partial w}{\partial Q}. \tag{4.1}$$

In general, WTP per QALY is not constant but depends on wealth, total QALYs, and risk posture with respect to longevity. The first term in equation (4.1) represents the pure WTP for improvements in health and longevity. The second term represents the feedback effect of changes in health and longevity on lifetime wealth. The magnitude of $\frac{\partial w}{\partial Q}$ may depend on whether QALYs are gained by improvements in health or increases in longevity. For example, Meltzer (1997) and Bleichrodt and Quiggin (1999) assume lifetime wealth depends on longevity but not on health. In the remainder of this section, I focus on pure WTP for health and longevity. Setting $\frac{\partial w}{\partial Q} = 0$, equation (4.1) simplifies to

$$V = -\frac{dw}{dQ} = \frac{rQ^{r-1} a(w)}{Q^r a'(w) + b'(w)}. \tag{4.2}$$

Consider a few special cases. First, let the individual be risk neutral with respect to longevity (r = 1, the case usually assumed in applications). Equation (4.2) simplifies to

$$V = \frac{a(w)}{Qa'(w) + b'(w)} \tag{4.3}$$

and so there is diminishing marginal WTP per QALY, so long as a'(w) > 0.[8]

In the case where the individual is indifferent to the level of his bequest (b'(w) = 0), equation (4.2) becomes

$$V = \frac{r}{Q}\frac{a(w)}{a'(w)}. \tag{4.4}$$

Again, WTP per QALY decreases with total QALYs (b'(w) = 0 implies a'(w) > 0).[9] In addition, WTP increases with r. Since the individual's degree of relative risk aversion with respect to longevity is equal to 1 – r, an increase in r decreases risk aversion (or increases risk proneness) with respect to longevity, so WTP per QALY depends on the risk posture for longevity. Specifically, an increase in risk aversion with respect to longevity decreases WTP per QALY.[10]

The second factor in equation (4.4) is the reciprocal of "boldness," also called the "fear of ruin." Boldness represents the individual's willingness to risk financial ruin in exchange for a marginal increase in wealth (Aumann and Kurz, 1977). WTP per QALY decreases with boldness.

Consider the general case given by equation (4.2). In the following subsections I investigate the dependence of WTP per QALY on wealth, health, longevity, and risk posture with respect to longevity.

Effect of Wealth

The effect of wealth on WTP per QALY can be examined by differentiating equation (4.2) with respect to wealth to obtain

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[8] If a'(w) < 0, WTP per QALY increases with QALYs and if a'(w) = 0, WTP per QALY is independent of total QALYs.

[9] Equations (4.3) and (4.4) are generalizations of Bleichrodt's and Quiggin's (1999) expression for WTP per QALY (assuming no effect on income) which requires b'(w) = 0 and r = 1.

[10] This result contrasts with the common but erroneous claim that risk aversion increases VSL (Eeckhoudt and Hammitt, 2002).

$$\frac{\partial V}{\partial w} = \frac{rQ^{r-1}\{a'[Q^r a' + b'] - a[Q^r a'' + b'']\}}{[Q^r a' + b']^2} \tag{4.5}$$

where w is suppressed to simplify notation. Further algebraic manipulation yields

$$\frac{\partial V}{\partial w} = \frac{u_Q}{u'}\left[\frac{a'(w)}{a(w)} + \pi(w)\right] \tag{4.6}$$

where u_Q and u' are the partial derivatives of utility (2.4a) with respect to Q and w, respectively, and π(w) is the measure of local risk aversion (equation (3.5)). Equation (4.6) shows that the effect of wealth on V depends on the ratio of the marginal utilities of QALYs and of wealth, and on the sum of the local boldness coefficient for a(w) and the local risk aversion with respect to wealth.

If a'(w) > 0 and the individual is weakly risk averse with respect to wealth for all values of Q, then π(w) is non-negative and V increases with wealth. V can decrease with wealth only when a"(w) or b"(w) is sufficiently large that π(w) becomes negative (i.e., the individual is risk seeking with respect to wealth). This result is analogous to the result for WTP to reduce mortality risk, where the marginal WTP to reduce mortality risk (the "value per statistical life" or VSL) increases with wealth except when the individual is sufficiently risk seeking in the states of life or death (Weinstein et al., 1980). In contrast to the case of WTP to reduce mortality risk, WTP per QALY can decline with wealth if a'(w) < 0 (regardless of financial risk posture). If a'(w) = 0 and the individual is risk neutral with respect to wealth, WTP per QALY is independent of wealth. The finding that, under reasonable conditions, WTP per QALY increases with wealth is unsurprising and has been previously recognized (e.g., Gold et al., 1996; Garber and Phelps, 1997).

Effects of Health and Longevity

To evaluate the dependence of V on health and longevity, differentiate equation (4.2) with respect to Q to obtain

$$\frac{\partial V}{\partial Q} = \frac{Q^{2(r-1)} ra[-a' + (r-1)b']}{[Q^r a' + b']^2}. \tag{4.7}$$

The sign of equation (4.7) depends on the risk posture with respect to longevity and the marginal utility of bequests. If a' > 0 and either the individual is risk neutral or risk averse with respect to longevity ($r \leq 1$) or the marginal utility of bequest is zero ($b' = 0$), then the marginal WTP per QALY decreases with increasing QALYs. In contrast, V may increase with Q if the individual is risk seeking with respect to longevity ($r > 1$) and assigns positive marginal value to his bequest ($b'(w) > 0$), or if $a' < 0$. Note that the sign of equation (4.7) is independent of Q, which implies that marginal WTP per QALY cannot increase for some values of $Q > 0$ and decrease for others. For states equivalent to death (where $q(h) = 0$), marginal changes in health and longevity have no effect on WTP per QALY.

Some empirical studies suggest that WTP per QALY decreases with QALYs. Krupnick et al. (2002) report evidence from a contingent valuation study which suggests that people with cancer are willing to pay more to reduce mortality risk than people without cancer. Smith et al. (2001) estimate VSL by analyzing compensating wage differentials among older workers and find that events which reduce health and or decrease life expectancy (e.g., developing angina or cancer) increase VSL. Since these factors reduce future QALYs, WTP per QALY must increase.

Effect of Risk Posture with respect to Longevity

The effect of risk posture with respect to longevity on V is ambiguous, as can be seen by differentiating equation (4.2) with respect to r to obtain

$$\frac{\partial V}{\partial r} = \frac{aQ^{2(r-1)}}{\left[Q^r a' + b'\right]^2}\left\{\left[Q + (r-1) - r^2\right]a' + \left[Q^{1-r} + (r-1)Q^{-r}\right]b'\right\} \qquad (4.8)$$

where the argument w is suppressed to simplify notation. For $r \leq 1$, $(r-1) - r^2 < 0$ so the sign of the term multiplying a' is less than zero for Q near zero and greater than zero for large values of Q. For $r > 1$, the sign of $(r-1) - r^2$ is negative and the sign of the term multiplying b' is positive. In either case, the sign of the entire expression is ambiguous.

5. Willingness to Pay to Reduce Mortality Risk

WTP to reduce mortality risk, described as the “value per statistical life” (VSL), is the marginal rate of substitution between wealth and the probability of surviving the current time period. The individual’s expected utility is given by

$$EU = (1 - p)\, u_a(w) + p\, u_d(w), \qquad (5.1)$$

where p is the probability of dying in the current period and $u_a(w)$ and $u_d(w)$ represent utility of wealth conditional on surviving and not surviving the period, respectively (e.g., Jones-Lee, 1974; Weinstein et al., 1980). Total differentiation of equation (5.1) yields

$$VSL = \frac{dw}{dp} = \frac{u_a(w) - u_d(w)}{(1-p)u_a'(w) + pu_d'(w)} = \frac{\Delta u(w)}{Eu'(w)}. \tag{5.2}$$

It is conventionally assumed that survival is preferred to death,

$$u_a(w) > u_d(w), \tag{5.3a}$$

(at least for $w > w_s$ where w_s is a subsistence wealth level), and that the marginal utility of wealth is greater given survival than death, and non-negative in the event of death,

$$u_a'(w) > u_d'(w) \geq 0. \tag{5.3b}$$

Under assumptions (5.3a–5.3b), VSL is positive and increasing in the risk level p (the "dead-anyway" effect; Pratt and Zeckhauser, 1996). Adding the additional assumption of weak risk aversion with respect to wealth,

$$(1 - p)\, u_a'' + p\, u_d'' \leq 0, \tag{5.3c}$$

is sufficient for VSL to increase with wealth.

In the standard model (equation (5.2)), the effects of health and longevity on VSL are ambiguous. The utility of surviving the current period $u_a(w)$ implicitly depends on health and longevity in the event of survival. Better health and greater life expectancy increase the utility of survival $u_a(w)$, and they may also increase the marginal utility of survival $u_a'(w)$. Reductions in life expectancy and health clearly limit the opportunities for gaining utility from wealth, and there is some empirical evidence that impaired health reduces the marginal utility of wealth (Sloan et al., 1998; Viscusi and Evans, 1990). Depending on the magnitudes of the effects of health and longevity on both the total and the marginal utilities of wealth given survival, better health and increased life expectancy may increase, decrease, or not affect VSL.

For the admissible utility function (2.4a), $u_a(w) = Q^r a(w) + b(w)$ and $u_d(w) = b(w)$. Substituting these expressions into equation (5.2) yields

$$VSL = \frac{dw}{dp} = \frac{Q^r a(w)}{(1-p)Q^r a'(w) + b'(w)}. \tag{5.4}$$

Because $u_a(w) = Q^r a(w) + b(w)$ and $u_d(w) = b(w)$ satisfy the standard assumptions (5.3a–5.3b), VSL is positive and increasing in mortality risk p (for Q > 0), and weak risk aversion with respect to wealth (assumption (5.3c)) remains a sufficient condition for VSL to increase with wealth.[11]

The effects of health and longevity on VSL can be seen by inspection of equation (5.4). If the individual is indifferent to the level of his bequest (b'(w) = 0), then VSL = a(w)/[(1-p) a'(w)] and is independent of health and longevity. Alternatively, if the marginal utility of the bequest is positive (b'(w) > 0), then the proportionate effect of an increase in Q is larger in the numerator than in the denominator of equation (5.4), and so VSL increases with health and longevity. These results can be verified by differentiating equation (5.4) to obtain

$$\frac{\partial}{\partial Q} VSL = \frac{rQ^{r-1}ab'}{\left[(1-p)Q^r a' + b'\right]^2} . \tag{5.5}$$

The value of equation (5.5) is zero when b'(w) = 0, and positive when b'(w) > 0. The admissible utility functions constrain the relationship between the effects of health and longevity on the utility and the marginal utility of wealth in such a way as to remove the ambiguity in the standard model (equation (5.2)). As noted above, Krupnick et al. (2002) and Smith et al. (2001) report empirical evidence suggesting that reduced health increases VSL, which is inconsistent with the admissible utility functions.

The effect of risk posture with respect to longevity on VSL can be examined by differentiating equation (5.2) with respect to r to obtain

$$\frac{\partial}{\partial r} VSL = \frac{ab'Q^r \log(r)}{\left[(1-p)Q^r a' + b'\right]^2} . \tag{5.6}$$

If the individual is indifferent to the level of his bequest (b'(w) = 0), then VSL is independent of his risk posture with respect to longevity. If the marginal utility of the bequest is positive (b'(w) > 0), then the effect of risk posture with respect to longevity depends on the risk posture. Recall that the measure of relative risk aversion with respect

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[11] The variable L implicit in equation (5.4) is most naturally interpreted as remaining longevity conditional on surviving the current period. As noted in footnote 5, under this interpretation of L, the admissible utility functions may be dynamically inconsistent.

to longevity is 1 – r, so an increase in r decreases risk aversion (increases risk proneness). If the individual is risk averse with respect to longevity (r < 1), then an increase in risk aversion (decrease in r) increases VSL. If the individual is risk seeking (r > 1), then an increase in risk proneness (increase in r) increases VSL. In the standard case where the individual is risk neutral (r = 1), a marginal change in risk posture with respect to longevity has no effect on VSL.

The ambiguous effect of a change in risk posture with respect to longevity on VSL is similar to the ambiguous relationship between a change in risk aversion with respect to wealth and VSL. An increase in risk aversion with respect to wealth may increase, decrease, or not alter VSL (Eeckhoudt and Hammitt, 2002).

6. Conclusion

If an individual's utility function for health and longevity can be represented by the number of risk-adjusted QALYs, and if his tradeoff between health and longevity is independent of wealth (as is routinely assumed), then his utility function for health, longevity, and wealth is tightly constrained. Several of the utility functions used in the literature are inconsistent with these assumptions (e.g., the monetary-loss-equivalent formulation, equation (1.4)). Significantly, utility functions that are additive over time (equation (1.2)) are inconsistent with these assumptions, except in the special case where the individual is risk-neutral with respect to longevity (r = 1) and the discount rate is zero (i.e., the discount factor $\delta = 1$).[12]

Under the admissible utility functions, the risk posture for longevity is independent of wealth, and may exhibit constant relative risk aversion, risk neutrality, or constant relative risk proneness. Under the standard assumption that the marginal utility of wealth is greater in life than as a bequest, the marginal utility of wealth is non-decreasing in health and longevity. The risk posture for wealth is not tightly constrained,

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[12] Additive utility models with non-zero discounting functions could be consistent with an alternative form of QALYs suggested by Johannesson et al. (1994), obtained by replacing longevity with the present value of discounted longevity (using the appropriate discount function) in the Pliskin et al. (1980) conditions. Risk neutrality over these alternative QALYs would be required.

but under the additional reasonable assumption that the individual is strictly risk averse for wealth while living and weakly risk averse with respect to bequests, the degree of financial risk aversion is non-decreasing in health and longevity.

Individual willingness to pay per QALY is not constant, but depends on health, longevity, and wealth. Assuming the marginal value of wealth in life exceeds its value as a bequest, there is diminishing WTP per QALY except when the individual is risk seeking with respect to longevity and the marginal utility of bequest is large. Under reasonable assumptions, WTP per QALY increases with wealth.

Under the standard model of willingness to pay to reduce mortality risk (the "value per statistical life" or VSL), the effects of health and longevity on WTP are ambiguous. The admissible utility functions remove this ambiguity, yielding the result that WTP to reduce mortality risk increases with health and longevity. WTP to reduce mortality risk depends on the risk posture with respect to longevity, with increasing departures from risk neutrality (either greater risk aversion or greater risk proneness) increasing VSL.

The result that individual WTP per QALY is not constant implies that cost-effectiveness analysis using QALYs is inconsistent with standard welfare economics and benefit-cost analysis. Moreover, WTP to reduce mortality risk is not proportional to life expectancy, and so an individual's value per statistical life year (VSLY) is not constant but depends on wealth, health, and life expectancy. Estimates of WTP to avoid mortality or morbidity calculated using a constant WTP per life year or per QALY are consequently inconsistent with the economic theory underlying WTP and QALYs. Cost-effectiveness analysis is more appropriately viewed as an alternative (Williams, 1993) rather than an implication of benefit-cost analysis.

References

Aumann, R.J., and M. Kurz, "Power and Taxes," *Econometrica* 45: 1137-1161, 1977.

Barksy, R.B., F.T. Juster, M.S. Kimball, and M.D. Shapiro, "Preference Parameters and Behavioral Heterogeneity: An Experimental Approach in the Health and Retirement Study," *Quarterly Journal of Economics* 112: 537-580, 1997.

Bleichrodt, H., and J. Quiggin, "Life-Cycle Preferences over Consumption and Health: When is Cost-Effectiveness Analysis Equivalent to Cost-Benefit Analysis?" *Journal of Health Economics* 18: 681-708, 1999.

Bleichrodt, H., P. Wakker, and M. Johannesson, "Characterizing QALYs by Risk Neutrality," *Journal of Risk and Uncertainty* 15: 107-114, 1997.

Corso, P.S., and J.K. Hammitt, "Preferences for Longevity Risks: Effects of Lottery and Personal Characteristics," Harvard School of Public Health, Boston, 2001.

Cutler, D.M., and E. Richardson, "Measuring the Health of the U.S. Population," *Brookings Papers Microeconomics* 1997: 217-271, 1997.

Dolan, P., "The Measurement of Health-Related Quality of Life for Use in Resource Allocation Decisions in Health Care," 1724-1760 in *Handbook of Health Economics*, Volume 1, A.J. Culyer and J.P. Newhouse, eds., Elsevier Science B.V., 2000.

Drèze, J., "L'Utilitè Sociale d'une Vie Humaine," *Revue Française de Recherche Opèrationelle* 6: 93-118, 1962.

Eeckhoudt, L.R., and J.K. Hammitt, "Does Risk Aversion Increase the Value of Mortality Risk?" Harvard School of Public Health, Boston, 2002.

Evans, W.N., and W.K. Viscusi, "Estimation of State-Dependent Utility Functions using Survey Data," *Review of Economics and Statistics* 73: 94-104, 1990.

Garber, A.M., and C.E. Phelps, "Economic Foundations of Cost-Effectiveness Analysis," *Journal of Health Economics* 16: 1-31, 1997.

Gold, M.R., J.E. Siegel, L.B. Russell, and M.C. Weinstein, *Cost-Effectiveness in Health and Medicine*, Oxford University Press, Oxford, 1996.

Grossman, M., "On the Concept of Health Capital and the Demand for Health," *Journal of Political Economy* 80: 223-255, 1972.

Harvey, C.M., "The Reasonableness of Non-Constant Discounting," *Journal of Public Economics* 53: 31-51, 1994.

Johannesson, M., "The Relationship Between Cost-Effectiveness Analysis and Cost-Benefit Analysis," *Social Science and Medicine* 41: 483-489, 1995.

Johannesson, M., J.S. Pliskin, and M.C. Weinstein, "A Note on QALYs, Time Tradeoff, and Discounting," *Medical Decision Making* 14: 188-193, 1994.

Johnson, F.R., E.E. Fries, and H.S. Banzhaf, "Valuing Morbidity: An Integration of the Willingness-to-Pay and Health-Status Index Literatures," *Journal of Health Economics* 16: 641-665, 1997.

Jones-Lee, M.W., "The Value of Changes in the Probability of Death or Injury," *Journal of Political Economy* 82: 835-849, 1974.

Keeney, R. and H. Raiffa, *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*, John Wiley and Sons, New York, 1976 and Cambridge University Press, Cambridge, 1993.

Krupnick, A., A. Alberini, M. Cropper, N. Simon, B. O'Brien, R. Goeree, M. Heintzelman, "Age, Health, and the Willingness to Pay for Mortality Risk Reductions: A Contingent Valuation Survey of Ontario Residents," *Journal of Risk and Uncertainty* 24: 161-186, 2002.

Liu, J.-T., J.K. Hammitt, J.-D. Wang, and J.-L. Liu, "Mother's Willingness to Pay for Her Own and Her Child's Health: A Contingent Valuation Study in Taiwan," *Health Economics* 9: 319-326, 2000.

Meltzer, D., "Accounting for Future Costs in Medical Cost-Effectiveness Analysis," *Journal of Health Economics* 16: 33-64, 1997.

Moore, M.J., and W.K. Viscusi, "The Quantity-Adjusted Value of Life," *Economic Inquiry* 26: 369-388, 1988.

Ng, Y.-K., "The Older the More Valuable: Divergence Between Utility and Dollar Values of Life as One Ages," *Journal of Economics* 55: 1-16, 1992.

Pliskin, J.S., D.S. Shepard, and M.C. Weinstein, "Utility Functions for Life Years and Health Status," *Operations Research* 28: 206-224, 1980.

Pratt, J.W., and R.J. Zeckhauser, "Willingness to Pay and the Distribution of Risk and Wealth," *Journal of Political Economy* 104: 747-763, 1996.

Pratt, J.W., "Risk Aversion in the Small and in the Large," *Econometrica* 32: 122-136, 1964.

Shepard, D.S., and R.J. Zeckhauser, "Survival versus Consumption," *Management Science* 30: 423-439, 1984.

Sloan, F.A., W.K. Viscusi, H.W. Chesson, C.J. Conover, and K. Whetten-Goldstein, "Alternative Approaches to Valuing Intangible Health Losses: The Evidence for Multiple Sclerosis," *Journal of Health Economics* 17: 475-97, 1998.

Smith, V.K., H. Kim, and D.H. Taylor, "Do the 'Near' Elderly Value Mortality Risks Differently?" paper presented at the Association of Environmental and Resource Economists workshop, Bar Harbor, Maine, June 2001.

Tolley, G., D. Kenkel, and R. Fabian (eds.), *Valuing Health for Policy: An Economic Approach*, University of Chicago Press, Chicago, 1994.

Viscusi, W.K., and W.N. Evans, "Utility Functions that Depend on Health Status: Estimates and Economic Implications," *American Economic Review* 80: 353-374, 1990.

Weinstein, M.C., D.S. Shepard, and J.S. Pliskin, "The Economic Value of Changing Mortality Probabilities: A Decision-Theoretic Approach," *Quarterly Journal of Economics* 94: 373-396, 1980.

Williams, A., "Cost-Benefit Analysis: Applied Welfare Economics or General Decision Aid?" in *Efficiency in the Public Sector*, A. Williams and E. Giardina, eds., Edward Elgar, London, 1993.