Health insurance when loss severities differ

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Preliminary version

Abstract - With information asymetry between contracting parties, adverse selection may result. A separating equilibrium can achieve. When severity differences and health state are aconsidered, first best equilibrium can be attained. Equilibrium with positive profit, full coverage, and opposite Rothschild-Stiglitz cans be feasible.
Key words - Adverse selection, Loss severity, Health insurance.
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Insurance plays a central role in health markets. More than 81% of health care expenditures in the United State are paid for by insurance. Adverse selection can be a significant resource allocation problem in this market. It is a consequence of asymmetric information : in health insurance market it occurs when consumers have better information about their illness than insurance compagnies. Is this condition satisfied in reality ?

Marquis (1992)[1] has shown that insureds can accurately forecast their risk and that this forecast affects the purchase of insurance coverage. Browne (1992) has proved that individuals who are in excellent self-reported health are significantly less likely to buy more insurance. Cuttler and Zeckhauser (1997)[2] present empirical evidence on adverse selection in two group of employees : Harvard University and the Group Insurance Commission of Massachussetts. In both groups, adverse selection is a significant concern. For example, at Havard, the university's decision to contribute an equal amount to all insurance plans led to the disappearence of the most generous policy.

To keep our analysis of adverse selection in health insurance as realist as possible, we use a bidimensionnal utility function. In fact, this insurance system is fundamentally different from automobile insurance because risk of illness can reduce simultaneously wealth and health. So, individuals receive utility whichs depends on their wealth and their health.

In this framework, Selden (1998) examines the scope for welfare-improving risk adjustment by a government with limited information. Risk adjustment can potentially improve both equity and efficiency. In this article, the economy consists of a large number of individuals wit identical preferences, each of whom faces a binary risk of illness[3] and an exogeneous probability of illness. Individuals differ only from probability.

A common hypothesis in economics insurance is to consider that agents have identical preferences. Existing empirical works suggest that risk aversion is heterogenous and this heterogeneity is crucial in understanding demand for insurance (Chiappori, 2000) because the single crossing can be violated. Then, the Rothschild-Stiglitz equilibrium does not hold.

For health insurance, Wolfe and Goldeeris (1991) stressed a positive correlation between coverage and self-reported health. So health heteogene-

[1] using RAND Health Insurance Experiment data and making a comparison of plan choices with age/sex adjustments under various group-rating regimes.

[2] making analysis of different plans' member characteristics and expenses.

[3] In the event of illness, each consumer suffers from a loss of health, and receives a level of medical care and an out-of-pocket bill.

ity has to be investigated. We then introduce a sanitary dimension in the Rothschild-Stiglitz's model (1976), noticed by RS. Our purpose is to characterize equilibrium of health market insurance with adverse selection. This article considers a model in which insurance customers differ in more than one characteristic. Customers differ with respect to illness probability, health state and severity loss. We suppose that a consumer in bad health costs more for insurance in expectation that a consumer in good health.

With the third unobservable, Doherty and Jung (1993) show that first-best, full-information equilibria become feasible. With two unobservables[4], Smart (2000) shows that the single crossing condition does not hold. So types may be pooled in equilibrium or are separated by raising premia above actuarially fair levels.

In this article, we show that the results of Rothschild-Stiglitz do not always hold. When individuals differ from health state, the single crossing is violated. So, we can obtain separating equilibrium with positive profit. However, first-best solutions become feasible when the health state for one group differs sufficiently from the others. The third result is more interesting. A constrated Rothschild-Stiglitz equilibrium can appear. Bad-health agent obtains less than full insurance while good-health agent receives full insurance.

The plan of this article is as follows: Section 2 introduces the model and describes the game. Section 3 studies the self-selection constraints. Section 4 characterizes the equilibrium of the health insurance market when loss severities differ differ. Section 5 concludes.

1 The model

We consider an insurance market which a large finite number of agents m, facing mild health risk, ie medical spending restores a person to initial health. Individuals receive utility, U, which depends on their wealth R and their health S. In the absence of insurance, for an agent of risk class i with $i \in \{H, L\}$, expected utility $U_i(W, S)$ is given by :

$$U_i = p_i u(R - D_i, S_i - T + m) + (1 - p_i) u(R, S_i) \tag{1}$$

[4] wihch are accident probability and risk aversion.

with p_i the probability of illness, T the loss of health, D_i medical spending and m the benefit of treatment. The health risk is mild so

$$-T + m = 0 \tag{2}$$

We assume that utility is monotonically increasing and concave in health and wealth where U_i and U_{ij} denote the first and the second partial derivatives of U with respect to its i th and j th arguments respectively. So $U_1 > 0, U_2 > 0, U_{11} < 0, U_{22} < 0$. We will assume that absolute risk aversion of a financial risk is decreasing in respect to health and wealth. For a same wealth, a type L agent has a less risk aversion than a type H agent. Bad-health agent fears more financial risk because he wants to face medical spending.

Disease probability p_i, medical spending D_i and initial health S_i each take on two values in the population, $i \in \{L, H\}$, with $S_H < S_L$. The high-risk H or bad health's state and the low-risk L or good health's state are drawn, respectively with probability λ_H and λ_L, where $\lambda_H + \lambda_L = 1$. We assume that bad-health agents cost more in expectation than good-health agents. Hence, $p_L D_L < p_H D_H$. Three configurations are possible :

1) $p_L < p_H$ and $D_L < D_H$
2) $p_L < p_H$ and $\frac{p_L}{p_H} D_L < D_H < D_L$
3) $p_H < p_L$ and $D_L < \frac{p_H}{p_L} D_H$.

The parameters p_i, D_i, S_i are common knowledge to the policyholders and to the insurers but the latter do not know the risk type of any individual. We assume that audit costs are too high for risk type verification to be feasible. When audit costs are high, incentive contracts are efficient.

There exists a limited number[5] of insurance firms (N with $N < m$) that may potentially enter to the market. We assume that there are two type of insurers which differ from rationality. Compagnies which number is N_c maximise their profit and mutual firms (N_m) earn zero profit, with . A contract $C(\pi, q)$ earns expected profit per customers of type i

$$\Pi(C) = \pi - p_i q \tag{3}$$

We seek subgame perfect Nash equilibria of the following two-stage RS game. In the first stage insurance firms choose wether to enter the market and, if so, offer a single insurance policy to potential customers. In the second stage, individuals choose a single policy among those on offer.

[5] because insurance reglementation.

2 Self-selection constraints

The characterization of equilibrium for the model proceed in several steps. First, we have to find which self-selection constraints are binding at equilibrium. Next, I establish for the three cases the specification of equilibrium.

When information on individual risk characteristics is public, each insurance compagnies knows the risk type of each individual. The optimal individual contract $(p_i D_i, D_i)$ yields full insurance coverage for each type of risk. Firms are constrained to earn zero expected profits. Under asymmetrical information, in the RS' model, full information competitive contracts are not adequate to allocate risk optimally. But, when loss severities differ, Doherty and Jung (1993) proved that first-best solutions became feasible. In this case, the self-selection constraints are not binding. If the self-selection is binding, one group of agent is rationed while other group is is fully insured.

Under asymmetric information, self-selection of type i is not binding when i-risk types prefer (or are indifferent) full insurance under the i-risk contract to alternative contract offering full insurance to j-risk type. Thus, the constrainst is not binding when

$$\begin{aligned} & U(w - p_i D_i, S_i) \\ \geq \ & p_i u(w - p_j D_j + D_j - D_i, S_i) + (1 - p_j) u(w - p_j D_j, S_i) \end{aligned} \tag{4}$$

with $(i, j) \in (L, H)^2$.

Lemma 1 *The self-selection constraint of an i-risk agent is not binding when*
a) $p_i < p_j$
b) $p_i > p_j$ *and* $S_i < \overline{S_i}$ *excepted for* $D_H < D_L$
with $(i, j) \in (L, H)^2$ *and* $p_H D_H > p_L D_L$

Proof. Inequation (4) is only clear for H-agent for $p_L < p_H$ and $D_L < D_H$. It does not hold. For the others cases, we have to make an approximation without loss of generality. We can show this lemma by using a Taylor expansion around $(w - p_i D_i)$. After simplifications[6], this inequation can be written :

$$\begin{aligned} 0 \ \gtrapprox \ & (p_i - p_j) D_i \\ & + \frac{1}{2} \left\{ \begin{array}{c} p_i (1 - p_i)(D_i - D_j)^2 \\ +(p_i - p_j)^2 D_j^2 \end{array} \right\} \frac{u_{11}(w - p_i D_i, S_i)}{u_1(w - p_i D_i, S_i)} \end{aligned} \tag{5}$$

[6] We denote that $p_H D_H - p_L D_L - D_H + D_L = D_L(1 - p_L) - D_H(1 - p_H)$.

Two cases have to be considered.

Case 1. If $p_i < p_j$ this inequation is always true because $u_{11} < 0$. So, lemma a) holds.

Case 2. If $p_i > p_j$ this inequation is more complex. Specifically, we have to define the coefficient of absolute risk aversion of a financial risk :

$$R_A(c, S_i) = -\frac{u_{11}(w - p_i D_i, S_i)}{u_1(w - p_i D_i, S_i)} \tag{6}$$

where $c = w - p_i D_i$.

Equation (5) can be simplified to :

$$R_A(c, S_i) \gtrapprox \frac{2(p_i - p_j)D_j}{\left\{p_i(1-p_i)(D_i - D_j)^2 + (p_i - p_j)^2 D_j^2\right\}} \tag{7}$$

We can define a critical value $R_A^*(c, \overline{S_i})$ at which the constrainst just binds.

$$R_A^*(c, \overline{S_i}) \approx \frac{2(p_i - p_j)D_j}{\left\{p_i(1-p_i)(D_i - D_j)^2 + (p_i - p_j)^2 D_j^2\right\}} \tag{8}$$

This may be rewritten as

$$R_A(c, S_i) > R_A^*(c, \overline{S_i}) \tag{9}$$

As, we have assumed that the coefficient of absolute risk aversion of a financial risk is declining in health, $S_i < \overline{S_i}$ implies that this last expression holds.

So, lemma b) holds. ■

We can now study our three cases :

Case 1 : $p_H > p_L$ and $D_H > D_L$.

If $i = L$ and $j = H$ equation (5) is true because $(p_i - p_j) < 0$ and $u_{11} < 0$. (lemma a).

If $i = H$ and $j = L$ equation (5) is true if and only if $R_A(c, S_H) > R_A^*(c, \overline{S_H})$. Hence $S_H < \overline{S_H}$. (lemma b). For low values S_H, the two self-selection constrainst do not bind. Consequently, the first-best, full-information equilibrium will prevail.

Case 2 : $p_H > p_L$ and $D_L > D_H > \frac{p_L}{p_H} D_L$.

If $i = L$ and $j = H$ equation (5) is true because $(p_i - p_j) < 0$ and $u_{11} < 0$. (lemma a)

If $i = H$ and $j = L$ equation (4) is wrong because $D_H < D_L$ and $p_H D_H > P_L D_L$. The self-selection constraint of i-risk agent will always bind and a full-information equilibrium is precluded.

Case 3 : $p_L > p_H$ and $D_H > D_L$

If $i = H$ and $j = L$ equation (5) is true because $(p_i - p_j) < 0$ and $u_{11} < 0$. (lemma a).

If $i = L$ and $j = H$ equation (5) is true if and only if $R_A(c, S_L) > R_A^*(c, \overline{S_L})$. Hence $S_L < \overline{S_L}$. (lemma b). As case 1, for low values S_L with $S_L > S_H$, full-information equilibrium will prevail.

3 Equilibrium in the assurance market

When health state is unobservable, a difficulty arise : indifference curve may cross at twice. If separation equilibrium is possible, two cases appear. First, agents may prefer a policy with excessive premium (positive profit). Second, first-best contracts become feasible. We suppose that equilibrium exists[7]. Our study focuses on the equilibrium type.

Before characterizing equilibrium, we have to define equilibrium of this market. It is clear that the nature of equilibrium is function how individual firms anticipate the behavior of rivals. Each insurer takes the actions of its competitor as given.

Definition 2 *No contract in the equilibrium set makes negative expected profit.*

When indifference curve cross at twice, individual may prefer a policy with a smaller deductible and excessive premium. So, second property (there is no contract that can make a positive profit) of RS'equilibrium is precluded.

Under competition, firms are constrained to offer contract which yield the more utility to individual.

Lemma 3 *Insurance contract C_i is the solution to :*

$$\begin{aligned} C_i &= \arg\max \; U_i(C_i) \\ \textit{subject to } U_j(C_j) &\geq U_j(C_i) \qquad (10) \\ \Pi(C_i) &\geq 0 \qquad (11) \end{aligned}$$

with $\{i, j\} \in \{H, L\}^2$ and $i \neq j$.

[7] We assume that pooling equilibrium is not attractive for the two groups of risk.

Proof. Under competition, firms offer the best contract to i-risk agent under the constrainst of positive contract (equation 11). But, this contract has to verify self-selection constraint (equation 10). If j-risk agents prefer the contract C_i, this contract can make negative profit.

When $p_i < p_j$ a contract C_i with coverage (q_i) and which earn zero profit on i-risk type, makes negative profit if it is choosen by j-risk-type. Expected profit of insurance firm is :

$$\Pi(C_i) = \lambda_j(p_i q_i - p_j q_i) + \lambda_i(p_i q_i - p_j q_i) \tag{12}$$

$\Pi(C_i) < 0$ because $p_i < p_j$. ■

3.1 $p_L < p_H$ and $D_L < D_H$

The characterization of equilibrium relies on the single-crossing property. As it is well known, an important issue in adverse selection models is whether the Spence-Mirrlees single crossing condition holds true. The equilibrium (if it exists) is separating if the slope of the S_H type indifference curve is more steep than S_L (in the plane q, π). So, for a policy (π, q) with π premium and q coverage, we obtain :

$$\frac{(1-p_H)}{p_H}\frac{u_1(W-\pi, S_H)}{u_1(W-\pi+q-D_H, S_H)} < \frac{(1-p_L)}{p_L}\frac{u_1(W-\pi, S_L)}{u_1(W-\pi+q-D_L, S_L)} \tag{13}$$

A necessary condition is :

$$\frac{u_1(W-\pi, S_H)}{u_1(W-\pi+q-D_H, S_H)} < \frac{u_1(W-\pi, S_L)}{u_1(W-\pi+q-D_L, S_L)} \tag{14}$$

This last condition holds if

$$\frac{u_1(W-\pi, S_H)}{u_1(W-\pi+q-D_L, S_H)} < \frac{u_1(W-\pi, S_L)}{u_1(W-\pi+q-D_L, S_L)} \tag{15}$$

with $R_1 = W - \pi$ and $R_2 = W - \pi + q - D_L$
$R_1 > R_2$ implicates that $\frac{u_1(R,S_H)}{u_1(R,S_L)}$ is declining with R.
Hence

$$-\frac{u_{11}(R, S_L)}{u_1(R, S_L)} < -\frac{u_{11}(R, S_H)}{u_1(R, S_H)} \tag{16}$$

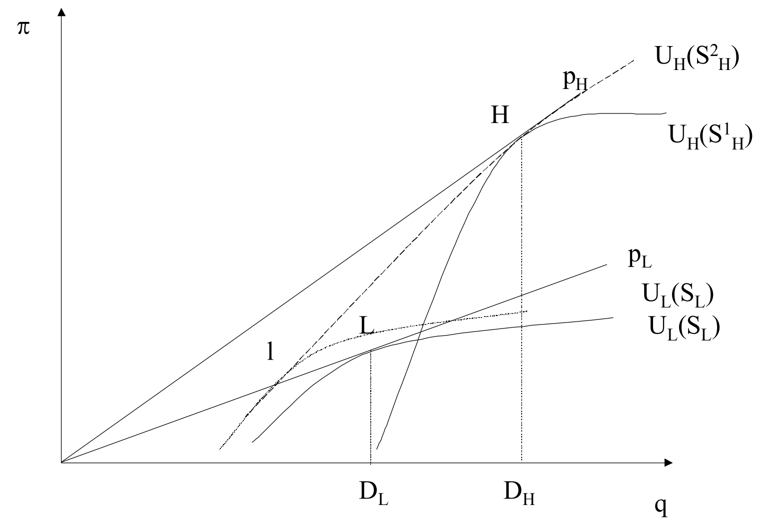

Figure 1: Equilibria when $p_L < p_H$ and $D_L < D_H$

This expression is true because we have assumed that the absolute risk aversion of a financial risk is declining with health ($S_H < S_L$). So the Spence-Mirrlees single crossing condition holds true. The equilibrium (if it exists) is separating and agents obtain insurance at fair price.

When $S_H = S_H^1 < \overline{S_H}$ every agent prefers his contract of full information (noticed L, H cf figure 1). Thus, information asymmetrics do not always cause inefficiencies in insurance and related markets. All agents receive full insurance : H and L contracts. In this case, we find for health insurance the Doherty-Jung results.

When $S_H = S_H^2 \geq \overline{S_H}$, H-risk type agent prefers L-contract than H-contract because it offers more consumption in each state. So the low-risk contract (noticed l cf figure 1) is defined by the self-selction of H-risk type. Agent in good health receive less than full insurance. The equilibrium is a standard Rothschild-Stiglitz equilibrium. Information's revelation is obtained by deductible. The H-risk type agent receives full insurance while L-risk obtains less than full insurance.This is stated in the following proposition.

Proposition 4 *When $p_L < p_H$ and $D_L < D_H$, the equilibrium :*
(a) if $S_H > \overline{S_H}$, is separating and type RS ;

(b) if $S_H \leq \overline{S_H}$, coincides with the full information equilibrium.

Proof. Lemma 1a indicates that self-selection constraint is never bind for the L-risk type. Lemma 1b implies that self-selection constraint of H-risk type is binding for high values of health state ($S_H > \overline{S_H}$). Two cases have to be considered.

a) if $S_H > \overline{S_H}$, H is obtained by maximising $U_H(C_H)$ subject to $\Pi(C_H) = 0$, and L by maximising $U_L(C_L)$ subject to $U_H(C_H) = U_H(C_L)$ and $\Pi(C_L) = 0$ (lemma 3) . We obtain $C_H = (p_H D_H, D_H)$ and $C_L = (p_L q_L, q_L)$ with $q_L < D_L$. So, H risk-type agent receives full insurance while L-risk type obtains less than full insurance. As the single crossing condition holds, equilibrium is separating and there is no contract outside the equilibrium set that can make a positive profit (for the proof, see RS).

b) if $S_H \leq \overline{S_H}$, all self-selection constraint are never bindind (lemma 1). i-risk contract is obtained by maximising $U_i(C_i)$ subject to $\Pi(C_i) = 0$ (lemma 3). We obtain $C_i = (p_i D_i, D_i)$. All agents obtain full insurance at fair price.

In these two cases, insurance firms (compagnies or mutual firms) are indifferent between offering L and H contract. They offer one of these contracts. ■

3.2 $p_L < p_H$ and $D_H < D_L$

In this case, the L type agent's incentive constraint is not binding. However, the H-risk type agent's constraint is binding. So the L-contract is defined by this constraint. In the standard explanation of screening in insurance markets, firms use deductibles to sort bad and good risk. In the present health's context firms' problems are more complicated. Preferences of agents can be double-crossing. Premiums which earn excessive profit arise in equilibrium when preferences are not single-crossing (Pannequin, 1992, Villeneuve, 1996 and Smart, 2000). It may be unfeasible for insurers to reduce premiums without violating incentive constraints.

A preliminary step in describing equilibrium is to characterize the relative position of the slope of the indifference curves' agents. To this end, let $C = (\pi, q)$ be a contract. We have to compare the slope of the indifference curves, ie $\frac{(1-p_H)}{p_H} \frac{u_1(W-\pi,S_H)}{u_1(W-\pi+q-D_H,S_H)}$ at $\frac{(1-p_L)}{p_L} \frac{u_1(W-\pi,S_L)}{u_1(W-\pi+q-D_L,S_L)}$.

If for $S_L = S_H$ $\frac{u_1(W-\pi,S_H)}{u_1(W-\pi+q-D_H,S_H)} < \frac{u_1(W-\pi,S_H)}{u_1(W-\pi+q-D_L,S_H)}$, then it is true for $S_L > S_H$ because $\frac{\partial\left[\frac{u_1(R_2,S)}{u_1(R_1,S)}\right]}{\partial S} < 0$. The equilibrium is closed to standard

Rothschild-Stiglitz equilibrium.

If for $S_L = S_H$ $\frac{(1-p_H)}{p_H}\frac{u_1(W-\pi,S_H)}{u_1(W-\pi+q-D_H,S_H)} > \frac{(1-p_L)}{p_L}\frac{u_1(W-\pi,S_H)}{u_1(W-\pi+q-D_L,S_H)}$, then it does not always hold for $S_L > S_H$ because the indifference curve of the H-risk type is declining with S_H. Then three cases must be considered.

1) The equilibrium (if it exists) is separating with zero profit if the slope of the S_H type indifference curve is more steep than S_L for the L_1-contract $(p_L q_L, q_L)$ with $q_L < D_H$. The critical value of S_H is defined when the slopes of the two type's agent are tangent with this contract. Set $\underline{S_H}$ this value. It verifies the follow expression :

$$\frac{(1-p_H)}{p_H}\frac{u_1(W-\pi,\underline{S_H})}{u_1(W-p_L q_L+q_L-D_H,\underline{S_H})} = \frac{(1-p_L)}{p_L}\frac{u_1(W-\pi,S_L)}{u_1(W-p_L q_L+q_L-D_L,S_L)} \tag{17}$$

Thus for $S_H \leq \underline{S_H}$ the equilibrium is separating (cf figure 2a).

2) For $\underline{S_H} < S_H < \overline{S_H}$, the indifferences' curve are tangent for a deductible q_L^1 with $q_L < q_L^1 < D_L$. With this profitable L_2-contract $(p_L q_L^1 + s, q_L^1)$ L-risk utility is higher than utility with the contrat $(p_L q_L, q_L)$ with s positive profit (cf figure 2b). Firms offer only profitable policy for L-risk and mutual firms policy with zero profit for H-risk. The equilibrium is separating and good-health agent pays an excessive premium for a deductible.

When S_H is incresing, the indifference's curve is less concave. The tangent point is shifted to the right. A critical value $(\overline{S_H})$ is reached when [8]:

$$\frac{(1-p_H)}{p_H}\frac{u_1(W-\pi,\overline{S_H})}{u_1(W-p_L D_L+D_L-D_H,\overline{S_H})} = \frac{(1-p_L)}{p_L} \tag{18}$$

3) For $S_H \geq \overline{S_H}$, the slope of the H-risk indifference curve is higher than L slope for the L_3-contract $(p_L D_L + s, D_L)$ (cf figure 2c). The tangent point is shifted to the right of D_L. According to the lack of above-insurance the pooling contract is not reached. H-risk type can receive at most D_H. Thus, every agent receive full insurance but good-health agent pays an excessive premium. Then Firms use excessive premium to sort bad and good health risk.

[8] We assume that $\overline{S_H} < S_L$.

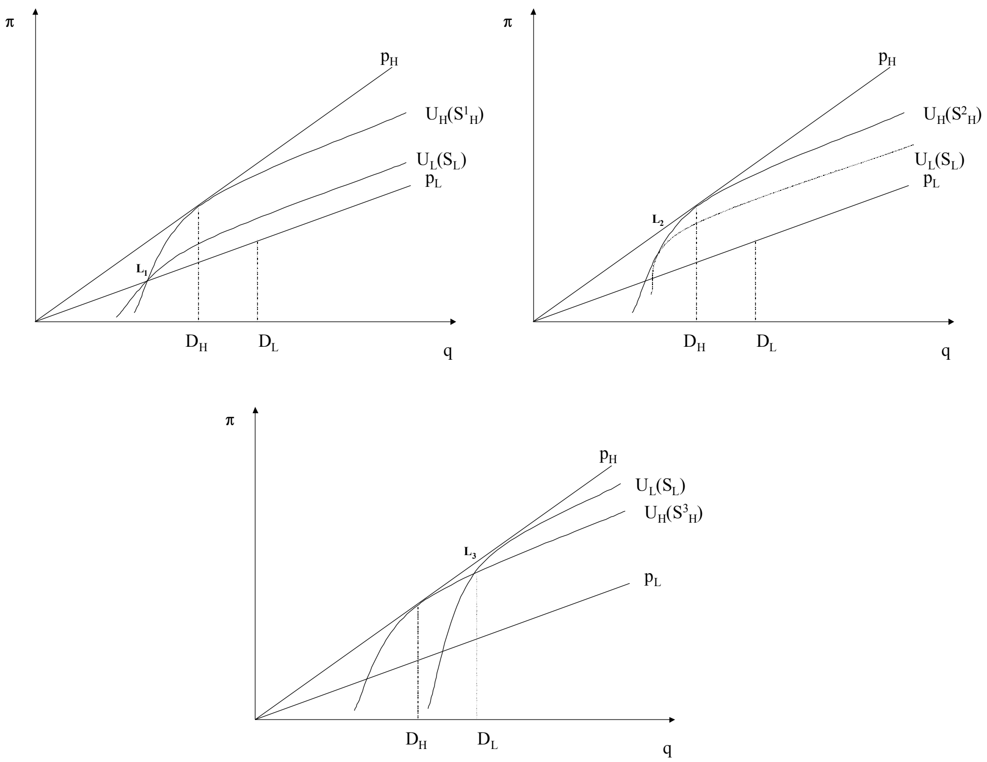

Figure 2: Equilibria when $p_L < p_H$ and $D_H < D_L$ - Right : 2a ; Left : 2b ; Middle ; 2c

The following proposition describes more precisely how the nature of equilibrium changes as health state increases.

Proposition 5 *When $p_L < p_H$ and $D_H < D_L$, if a separating equilibrium exists, then a bad-health agent obtains full insurance with fair price and*

(a) if $S_H \leq \underline{S_H}$, then good-health receives less than full insurance with fair price.

(b) if $\underline{S_H} < S_H < \overline{S_H}$, then good-health agent receives less than full insurance and pays an excessive premium.

(c) if $S_L > S_H \geq \overline{S_H}$, then good-health agent receives full insurance and pays an excessive premium.

Proof. Lemma 1 indicates that the L type agent's incentive constraint is never binding and the H-risk type agent's constraint always binding. So H is obtained by maximising $U_H(C_H)$ subject to $\Pi(C_H) = 0$, and L by maximising $U_L(C_L)$ subject to $U_H(C_H) = U_H(C_L)$ and $\Pi(C_L) \geq 0$ (lemma 3). We obtain $C_H = (p_H D_H, D_H)$. As the single crossing does not hold, $\Pi(C_L)$ may be positive. Hence, the characterization of L contract depends on the relative position of the indifference curves. Three cases have to be considered.

a) When $S_H = S_H^1$ with $S_H^1 \leq \underline{S_H}$, the H-indifference curve is more concave. For L_1-contract $(p_L q_{L1}, q_{L1})$ with $q_{L1} < D_L$, the slope of the H-risk indifference curve is higher than L slope (cf. figure 2a). Equilibrium is separating with zero profit ($\Pi(C_L) = 0$). Insurance firms (compagnies or mutual firms) are indifferent between offering L and H contract. They offer one of those contracts.

b) When $S_H = S_H^2$ with $\underline{S_H} < S_H^2 < \overline{S_H}$, the H-indifference curve becomes less concave. For L_2-contract (π_{H2}, q_{L2}), solution to lemma 3, with $\pi_{H2} > p_L q_{L2}$ and $q_{L1} < q_{L2} < D_L$, the slope of the H-risk indifference curve is equal to the L-slope (cf. figure 2b) . L type strictly prefers L_2-contract, an incentive-compatible contract which lower deductible and higher premium than L_1-contract. In this case, $\Pi(C_L) > 0$. Equilibrium is separating. Good-health agent receives less than full insurance and pays an excessive premium. Compagnies firms offer L-contract and mutual firm H-contract.

c) When $S_H = S_H^3$ with $S_L > S_H^3 \geq \overline{S_H}$, the slope of H-indifference curve becomes less and less concave. For L_3-contract (π_{L3}, D_L), solution to lemma 3, with $\pi_{L3} > p_L D_L$, the slope of the H-risk indifference curve is less than the L-slope (cf. figure 2c) . L type strictly prefers L_3-contract, an incentive-compatible contract which full coverage and higher premium than L_2 or L_1-contract. In this case, $\Pi(C_L) > 0$. Equilibrium is separating. Every agent obtains full insurance but good-health agent and pays an excessive premium. Compagnies firms offer L-contract and mutual firm H-contract.
■

To conlude, information's revelation is obtained by deductible and or excessive premium. The L-risk type agent receives less than full insurance and can pay an excessive premium.

3.3 $p_H < p_L$ and $D_L < D_H$

In this case, the H-risk type agent's constraint is never binding. However, the L-risk type agent's self-selection constraint can be binding. So the H-contract can be defined by this constraint. Can constrasted Rothschild-Stiglitz equilibrium appear?

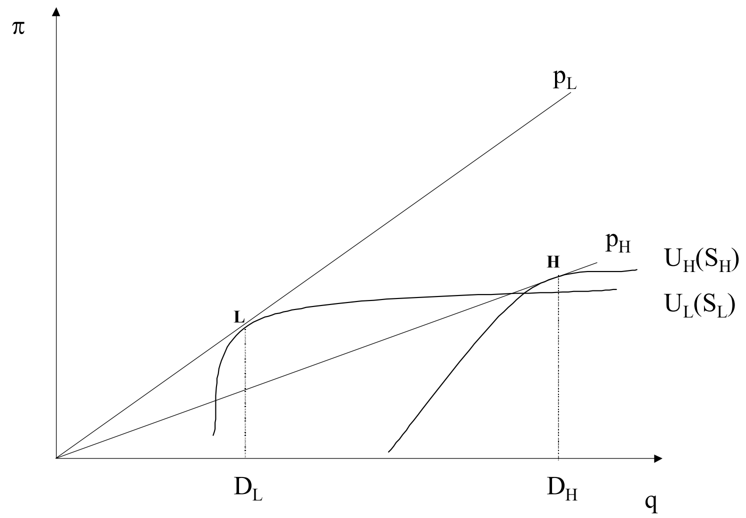

Figure 3: First-best equilibrium

In the present health's context firms' problems are more complicated. First, equilibrium can be full-information. Second, rreferences of agents can be double-crossing. Excessive premiums can be arised in equilibrium. It may be unfeasible for insurers to reduce premiums without violating incentive constraints. Four equilibrium configurations are possible.

When $S_H < S_L < \underline{S_L}$, any self-selection constraint is binding. Every agent prefers his contract of full information (noticed L, H cf figure 3). Thus, information asymmetrics do not always cause inefficiencies in insurance and related markets. All agents obtain full insurance.

When $S_L > \underline{S_L}$, the L-risk type agent's self-selection constraint is binding. A preliminary step in describing equilibrium is to characterize the relative position of the slope of the indifference curves' agents. To this end, let $C = (\pi, q)$ be a contract. We have to compare the slopes of the indifference curves, ie $\frac{(1-p_H)}{p_H}\frac{u_1(W-\pi,S_H)}{u_1(W-\pi+q-D_H,S_H)}$ at $\frac{(1-p_L)}{p_L}\frac{u_1(W-\pi,S_L)}{u_1(W-\pi+q-D_L,S_L)}$. The L-indifference curve depends on S_L. It becomes less concave when S_L increases.

1) We assume that the parameters (p_H, p_L, D_H, D_L and S_H) are fixed like there exists a separating equilibrium exists. Thus, for H_1-contract $(p_H q_H, q_H)$,

$$\frac{(1-p_L)}{p_L}\frac{u_1(W-p_Hq_H,S_L)}{u_1(W-p_Hq_H+q_H-D_L,S_L)} < \frac{(1-p_H)}{p_H}\frac{u_1(W-p_Hq_H,S_H)}{u_1(W-p_Hq_H+q_H-D_H,S_H)} \quad (19)$$

This condition is true for some values of S_L. This equilibrium (if it exists) is separating with zero profit. The good-health agents receive full insurance while the bad-health agents obtain less than full insurance.

We can define a critical value of S_L is defined when the slopes of the two type's agent are tangent with this contract. Set $\underline{S_L^1}$ this value. It verifies the follow expression :

$$\frac{(1-p_L)}{p_L}\frac{u_1(W-p_Hq_H,\underline{S_L^1})}{u_1(W-p_Hq_H+q_H-D_L,\underline{S_L^1})}=\frac{(1-p_H)}{p_H}\frac{u_1(W-p_Hq_H,S_H)}{u_1(W-p_Hq_H+q_H-D_H,S_H)} \quad (20)$$

Thus, for $S_H < S_L \le \underline{S_L^1}$ the equilibrium is separating (cf figure 4a). There is a constrated Rothschild-Stiglitz equilibrium.

2) For $\underline{S_L^1} < S_L < \overline{S_L}$, the indifferences' curve are tangent for a deductible q_H^1 with $q_H < q_H^1 < D_H$. With this profitable H_2-contract $(p_Hq_H^1+s, q_H^1)$ H-risk utility is higher than utility with the contrat $(p_Hq_H^1, q_H^1)$ with s positive profit (cf figure 4b). Firms offered only profitable policy for H-risk and mutual firms policy with zero profit for L-risk. The equilibrium is separating and bad-health agent pays an excessive premium for a deductible.

When S_L is incresing, the indifference's curve is less concave. The tangent point is shifted to the right. A critical value $(\overline{S_L})$ is reached when :

$$\frac{(1-p_H)}{p_H}=\frac{(1-p_L)}{p_L}\frac{u_1(W-p_HqD_H+s,\overline{S_L})}{u_1(W-p_Hq_H+q_H-D_L+s,\overline{S_L})} \quad (21)$$

3) For $S_L \ge \overline{S_L}$, the slope of the L-risk indifference curve is higher than H slope for the L_3-contract (p_HD_H+s, D_H) (cf figure 4c). The tangent point is shifted to the right of D_H. According to the lack of above-insurance the pooling contract is not reached. L-risk type can receive at most D_L. Thus, every agent receive full insurance but bad-health agent pays an excessive premium. Then Firms use excessive premium to sort bad and good health risk.

The following proposition describes more precisely how the nature of equilibrium changes as health state increases.

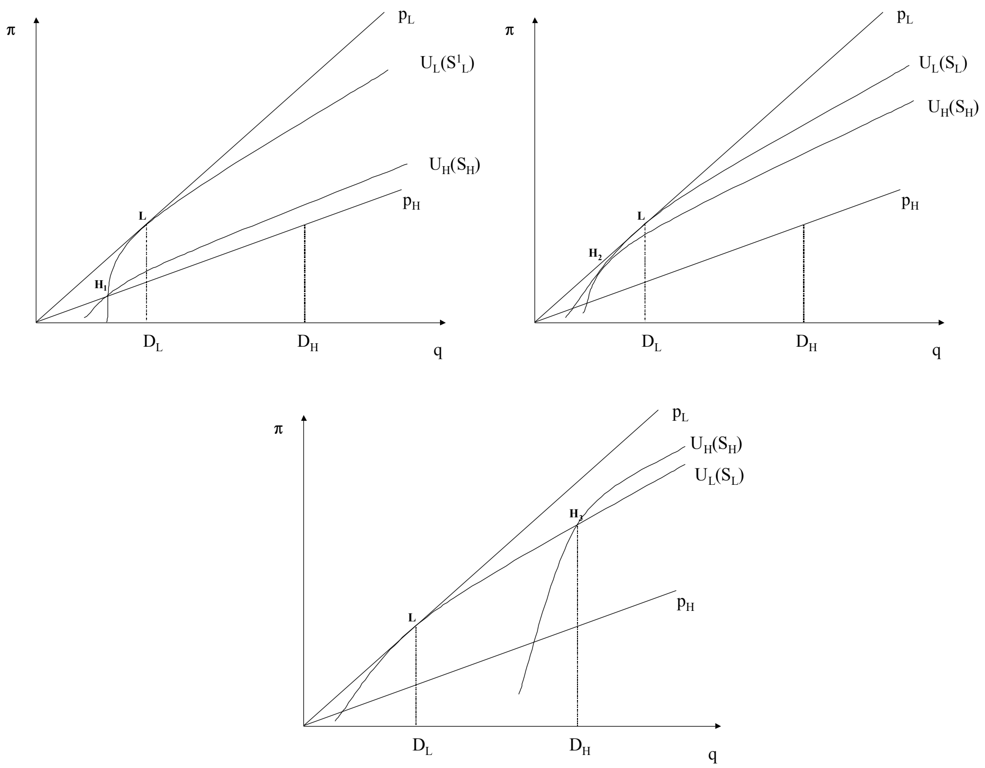

Figure 4: Equilibria when $p_H < P_L$ and $D_L < D_H$ - Left : 4a ; Right : 4b ; Middle : 4c.

Proposition 6 *When $p_H < p_L$ and $D_L < D_H$, if a separating equilibrium exists for S_L, then a good-health agent obtains full insurance at fair price and*

(a) if $SH < S_L \leq \underline{S_L}$, then bad-health receives full insurance at fair price.

(b) if $\underline{S_L} < S_L \leq \underline{S_L^1}$, then bad-health receives less than full insurance at fair price.

(c) if $\underline{S_L^1} < S_L < \overline{S_L}$, then bad-health agent receives less than full insurance and pays an excessive premium.

(d) if $S_L \geq \overline{S_L}$, then bad-health agent receives full insurance and pays an excessive premium.

Proof. For (a) Lemma 1 indicates that self-selection constraint is never bind for the i-risk type. i-contract is obtained by maximising $U_i(C_i)$ subject to $\Pi(C_i) = 0$ (lemma 3) . We obtain $C_i = (p_i D_i, D_i)$ (cf. figure 3).

Equilibrium coincides with the full information equilibrium.Insurance firms (compagnies or mutual firms) are indifferent between offering L and H contract. They offer one of those contracts.

For (b), (c) and (d) The L-self selection constraint holds (lemma 1). The insurance firms have to sort bad and good risk. Hence, the characterization of H contract depends on the relative position of the indifference curves. Three cases have to be considered.

(b) When $S_L = S_L^1$ with $\underline{S_L} < S_L^1 \leq \underline{S_L^1}$, the slope of L-indifference curve is more concave. For H_1-contract, (π_1, q_{H1}), solution to lemma 3, with $\pi_1 > p_H q_{H1}$ and $q_{H1} < D_H$, the slope of the L-risk indifference curve is higher than H slope (cf. figure 4a). Equilibrium is separating with zero profit $(\Pi(C_H) = 0)$. Insurance firms (compagnies or mutual firms) are indifferent between offering L and H contract. They offer one of those contracts.

(c) When $S_H = S_H^2$ with $\underline{S_L^1} < S_L^2 < \overline{S_L}$, the slope of L-indifference curve becomes less concave. For H_2-contract (π_{H2}, q_{H2}) solution to lemma 3, with $\pi_{H2} > p_H q_{H2}$ and $q_{H1} < q_{H2} < D_H$, the slope of the L-risk indifference curve is equal to the H-slope (cf. figure 4b). H type strictly prefers H_2-contract, an incentive-compatible contract which lower deductible and higher premium than H_1-contract. In this case, $\Pi(C_H) > 0$. Equilibrium is separating. Bad-health agent receives less than full insurance and pays an excessive premium. Compagnies firms offer H-contract and mutual firm L-contract.

(d) When $S_H = S_L^3$ with $S_L^3 \geq \overline{S_L}$, the slope of L-indifference curve becomes less and less concave. For H_3-contract (π_{H3}, D_H), solution to lemma 3, with $\pi_{H3} > p_H D_H$, the slope of the L-risk indifference curve is low than the H-slope (cf. figure 4c). H type strictly prefers H_3-contract, an incentive-compatible contract which full coverage and higher premium than H_2 or H_1-contract. In this case, $\Pi(C_H) > 0$. Equilibrium is separating. Every agent obtains full insurance but bad-health agent pays an excessive premium. Compagnies firms offer H-contract and mutual firm L-contract. ■

To conlude this section, information's revelation is obtained by deductible and or excessive premium. The H-risk type agent receives less than full insurance and can pay an excessive premium.

4 Conclusion

Health insurance depends on the characteristics of the buyer. People with a bad health cost more to insurers than those with a good health. When agents are privately informed about their health and their expected losses, Rothschild-Stiglitz equilibrium can not be applied. The addition of severity losses' difference and health change the nature of equilibrium in significant ways. First, our analysis shows the possibility that a first-best equilibrium could prevail. Second, premiums charged for insurance may exceed the actuarially fair rate that would be charged under full information. Third, when all agents obtain full insurance, excessive premiums can be necessary to screen for insured. Fourth, equilibruim can be constrated Rothschild-Stiglitz. Bad-health agent obtains less than full insurance while good-health agent receives full insurance.

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