Allocation of Capital in the Insurance Industry*

J. David CUMMINS
The Wharton School

1. Introduction

The purpose of this article is to provide an overview of the various techniques that have been suggested for allocating equity capital. Capital allocation in this context refers to the determination of the amount of a firm's equity capital that is assigned to each project or line of business undertaken by the firm. The objectives of capital allocation are discussed in more detail below. However, firms are usually concerned about capital allocation in the context of pricing and project selection, e.g., to determine the proportion of the firm's overall cost of capital that must be contributed by each line of business in order to maximize firm value. The discussion of capital allocation is conducted in the context of the insurance industry. However, most of the techniques discussed are perfectly general and can be applied in other industries as well.

Capital allocation is perhaps of special interest to financial firms such as insurers. For such firms, the principal providers of debt capital (insurance reserves) are also the firm's principal customers. Unlike the holders of bonds and other (non-insurance) debt capital, insurance policyholders cannot protect themselves against the insolvency of specific debt issuers by holding a diversified portfolio. Unlike the diversified bond investor, the typical policyholder relies upon one insurer (or at most a few, in the case of life insurance) for each type of protection purchased (e.g., auto insurance, homeowners insurance, health insurance, etc.). Most insurance policies are purchased not as an investment but to protect against adverse financial contingencies. Thus, insolvency risk plays a special role in the insurance industry, and capital is held to assure policyholders that claims will be paid even if larger than expected.

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In discussing capital allocation, it is important to keep in mind that the insurer's entire capital is available to pay the claims arising from any specific policy or line of business. If the insurer becomes insolvent it is the entire company that enters bankruptcy – the firm does not go bankrupt line by line. Nevertheless, it is often useful to think of capital as being allocated by line of business for pricing, underwriting, and other types of decision making. Taking a close look at capital allocation in the context of the insurance industry is also useful to elucidate the interaction between financial decision making and the risk-based capital rules applied to the insurance industry by regulators. Capital allocation is also related to recently emerging concepts such as *risk adjusted return on capital* (RAROC) and *economic value added* (EVA), which have become important management decision making techniques for both financial and non-financial firms.

Before beginning the discussion of capital allocation, two cautionary notes are in order: first, the terminology in this field is far from standardized. Therefore, a concept such as "risk-adjusted return on capital" that I will discuss is sometimes called by slightly different names in the literature. It is important to focus on the concepts and not on the nomenclature. Second, because this is an overview, it omits many important details that a firm would have to deal with to allocate capital in practice. Filling in the details to enable insurers to move from the concepts to practical applications in capital allocation provides a promising avenue for future research.

The remainder of the article is organized as follows: Section 2 discusses how capital allocation can be used to maximize the value of the firm. Section 3 first provides an overview of the capital allocation techniques that have been suggested and/or employed in practice and then provides a detailed discussion of the principal methods. Section 4 concludes.

2. Using Capital Allocation to Maximize Value

Why allocate capital? The motivation for anything a firm does should be to maximize shareholder value, that is, to increase the market value of equity capital. Unfortunately, this straightforward and powerful objective is often overlooked in practice. Through extensive discussions with people in the insurance industry, it appears that many firms are managing to their GAAP (generally accepted accounting principles) balance sheets and income statements, with the objective of showing healthy GAAP earnings and/or maximizing the value of GAAP equity. Of course, the firm needs to be cognizant of its GAAP performance because of the importance of accounting results to financial analysts and traders. However, it is a mistake to lose sight of the firm's true mission – the maximization of market value. Marking to market should play a critical role in the firm's internal decisions.

Capital allocation can be used to facilitate and improve the measurement of the economic profitability of businesses with different sources of risk and different capital requirements. In the insurance industry, it is customary to define businesses in terms of lines of insurance, for example, the commercial liability line or the auto liability line. Although this is the traditional approach, insurers need to step back and think carefully more about the issue of «what is a business?» in designing capital allocation and performance measurement systems.

Sometimes the banking literature talks about deposit accumulation or gaining demand deposits as one business, and making loans as another business. In this conceptualization, the economic concept of *transfer pricing* is used, whereby the bank's loan origination business will borrow money from the deposit accumulation business and pay an implicit rate of interest in return for having funds to invest in loans. This approach could also be used in insurance. The underwriting operation could be considered as a funds-generating business in which money is being borrowed from policyholders. The underwriting operation then would lend the funds to the investment business in return for a transfer price.

In either of the above lines of business concepts, the maturity and duration characteristics of the debt capital and the investments resulting from the insurer's different businesses must be recognized. Thus, funds generated by issuing long-tail liability policies are likely to lead to different investment objectives than funds raised by issuing short-tail property insurance policies in order to manage the risks of duration and convexity. One cannot assume that the long-term liability line has an asset portfolio that looks like the company; it has to be managed to meet the firm's overall objectives in terms of interest rate risk. Duration and convexity management is extremely technical. Therefore, special care must be exercised in giving a particular business credit for the money it generates, while, at the same time, charging it for the use of capital. This is the case for both the debt and equity capital needed to operate the business.

The primary link between capital allocation and value maximization is to enable the firm to measure performance by line of business to determine whether each business is contributing sufficiently to profits to cover its cost of capital and add value to the firm. To measure the cost of capital, it is necessary to determine the capital reqired to offer each type of insurance – relatively risky lines typically require more capital than less risky lines. For example, one might wonder whether the commercial liability insurance business is making an adequate profit, that is, whether the insurer should be charging higher or lower prices than at present, whether it should exit this business or perhaps devote more capital to it. Insurers can maximize value by shedding unprofitable businesses as well as by identifying profitable new projects. By withdrawing from unprofitable lines, the insurer may be able to increase the market value of equity, even as revenues decline. The ultimate objective should not be revenue growth, but maximizing net worth.

To provide a framework for the discussion of capital allocation methodologies, it is helpful to provide a simple mathematical statement of the capital allocation problem. Define x_i as the proportion of the firm's equity capital allocated to business i, where x_i is between 0 and 1. Thus, x_i indicates the proportion of capital which is allocated to business *i* and, therefore, the amount of capital allocated to business i is C_i, which is the total capital, C, multiplied by x_i. If the firm has N businesses, then

$$\sum_{i=1}^{N} x_i \leq 1 \quad and \quad \sum_{i=1}^{N} C_i \leq C$$

That is, the sum of the capital allocated to all of the firm's business will be less than or equal to the firm's total capital. While it may seem surprising that a firm may not assign all of its capital to its businesses, in fact some leading-edge researchers argue against allocating all of the capital and favor, instead, an allocation which results in less that 100 percent being assigned (Merton and Perold 1993). We return to this issue in the discussion below of the use of option models to allocate capital.

Once capital has been allocated by line, how can the resulting allocations be used to maximize firm value? One approach that has received considerable attention is to calculate the *risk-adjusted return on capital (RAROC).* RAROC is defined as the net income from a line, divided by the capital allocated to the line. That is,

$$RAROC_i = \frac{Net\ Income_i}{C_i}$$

where C_i is the capital allocated to line of business i.. The numerator of the RAROC formula, net income, also needs to be defined carefully. It may seem obvious, but it really is not once it is considered in economic terms. Basically, net income in the RAROC formula should be after taxes and interest expense. Even though interest expense is a banking term, it also applies to insurers in the form of underwriting loss. That is, the insurance market implicitly discounts the loss cash flows for the time value of money, meaning that the underwriting profit is negative in most cases. The negative underwriting return, which is analogous to interest expense, needs to be taken out when calculating the return from a line of business.

Once the RAROC for a line of business has been calculated, how does the firm know whether the line's current risk-adjusted return is adequate? The risk-adjusted return should be compared with the cost of capital for business i, where the cost of capital is obtained using an appropriate asset pricing model. If the risk-adjusted return equals or exceeds the cost of capital, then continuing to devote resources to this line of business is consistent with the goal value-maximization. However, if the risk-adjusted return is below the cost of capital, the line of business is reducing the firm's market value. In this circumstance, the firm should take some action to improve the situation such as re-pricing the insurance, tightening underwriting standards, or withdrawing from the line of business.

A slightly more formal way of determining whether a particular line of business is adding to firm value goes under the name of *economic value-added (EVA).*[1] Economic value-added measures the return on an investment in excess of its expected or required return. EVA seeks to identify lines that create value for the firm. EVA is net income minus the cost of capital, or hurdle rate, for a certain business, multiplied by the capital allocated to the business:

$$EVA_i = \text{Net Income}_i - r_i C_i$$

where r_i = cost of capital (hurdle rate) for business i. Thus, if EVA ≥ 0, writing the line of business is consistent with value maximization, while if EVA < 0, the line is destroying firm value. The EVA formula can be changed slightly to put the results in rate of return format, creating a measure called the *economic value added on capital (EVAOC).* EVAOC is defined as EVA divided by the capital allocated to a line, i.e.,

$$EVAOC_i = (\text{Net Income}_i / C_i) - r_i$$

This is similar to RAROC, except that the line's cost of capital is subtracted. Again, if EVAOC is positive, the line is creating value for the firm.

An important detail is how to determine the cost of capital for business i. This too can present a problem in the insurance industry due to data limitations. One approach proposed by finance researchers to estimating the cost of capital for a line of business is the «pure play» technique. The pure play approach estimates the cost of capital by finding other firms that offer only one line of business. The cost of capital for a business in the multiple-line firm can then be based on the cost of capital of mono-

[1] There are probably five or ten other concepts more or less similar to EVA that have been discussed in the literature. I use EVA as an example of these concepts.

line firms writing only that specific type of business. This approach is problematical in the insurance industry because it is difficult to find firms that write only one line of business. Even if such a firm or firms could be found, the underwriting risk characteristics of the pure play firms could differ significantly from those of a given line of business written by the multiple line firm. An alternative to the pure play technique is the use of so-called "full-information betas" to determine the cost of capital (Kaplan and Peterson 1998). This estimation technique uses data on conglomerates (firms that write several lines of business) to conduct regressions that permit the estimation of the cost of capital by line.

Another problem in the insurance industry is the lack of quality data. An insurer thinking about implementing VaR, EVA, and the other economic methodologies discussed here needs to think about revising its data system to capture the data required to implement the methodologies. Insurers should be designing data systems that allow them to report underwriting results frequently, for example, on a monthly basis. Data quality is crucial. With inadequate data, even a perfect model will fail. Most insurers do not have the necessary data to implement VaR, RAROC, and EVA at the present time.

3. Capital Allocation Techniques

Various methodologies have been developed that could provide the foundation for a system of capital allocation. The following is by no means an exhaustive list. Many other proposals can be found throughout the related literature. I first provide an overview of the methods and then go into more detail in discussing each method separately.

3.1. Overview of Capital Allocation Techniques

Regulatory Risk-Based Capital. In the United States, regulators have developed a formula to calculate the risk-based capital of insurers. A company's risk-based capital is used to define the minimum capital it must hold in order to avoid regulatory intervention. The regulatory thresholds or "action levels" are determined by the risk-based capital ratio, which is the ratio of the company's actual capital to its risk-based capital (see Cummins, Harrington, and Niehaus 1993). If the insurer's actual capital is greater than 200 percent of its risk-based capital, no regulatory action occurs. However, if actual capital falls below 200 percent of risk-based capital, various regulatory actions are taken, depending upon how far actual capital falls below 200 percent. An insurer's risk-based capital is computed by a formula designed to require more capital for riskier companies.

Some insurers actually use the regulatory risk-based capital formula in allocating capital for purposes of managing the firm. In my opinion, this is unwise, because the regulatory charges are of questionable accuracy and are based on book rather than market values.[2] Furthermore, the regulatory charges ignore important sources of risk such as interest rate (duration and convexity) risk, as well an the insurer's transactions in the derivatives market. Even if the charges were accurate, they would be accurate only for the average firm in the industry. Consequently, in the case of firms with books of business having above or below-average risk, the regulatory charges would produce inappropriate allocations of capital. The result is that businesses may be charged for too much or too little capital, leading to sub-optimal decisions.

The Capital Asset Pricing Model. The second approach involves using one of the oldest of modern financial theory technologies, the capital asset pricing model (CAPM). This is not the best solution to the problem, but it may provide a helpful benchmark based on a familiar methodology. At the very least, the use of the CAPM allows managers to compare the preferred method to the results generated by a classic technique.

Value At Risk (VaR). This value at risk concept has become very popular in the banking and investment banking communities where there is a need to examine the risk exposure of the firm's trading book for foreign exchange, bonds, etc. Simply stated, the *value at risk (VAR)* is the amount the firm could lose with a specified small probability, such as 1 percent, in a specified period of time. The measurement of value at risk from currency and securities trading has advanced rapidly, thanks in part to

[2] Cummins, Harrington, and Klein (1995) show that risk-based capital has low predictive power in terms of identifying companies that are likely to become insolvent.

daily and even more frequent data on exchange rates and asset prices which allow for very accurate and sophisticated calculations of VaR.[3]

VaR is also likely to be very useful to insurers and, in fact, is closely related to time honored insurance and actuarial concepts such as the probability of ruin and the maximum probable loss. Unfortunately, the application of the most sophisticated VaR techniques requires very frequent data (monthly data is an absolute minimum), but insurance prices and losses are not observed with sufficient frequency nor in a market context. Most insurers do generate such data internally. Using sophisticated tools such as Value at Risk requires an integration of the capital allocation methodology with data processing and information systems to ensure that pertinent and useful data are generated to provide inputs for the Value at Risk models.

Marginal Capital Allocation. Marginal capital allocation is a term that can be applied to the capital allocation technique proposed by Robert Merton and Andre Perold (1993) and to a related technique developed by Stewart Myers and James Read (1999). Both techniques are based on the option pricing model of the firm. In the options view of the firm, the value of the policyholders' claim on the firm is equal to the present value of losses minus the value of the *insolvency put option*. The insolvency put option is the expected loss to policyholders due to the possibility that the firm will default. The simplest option interpretation of the firm involves a one-period, two-date model where the firm issues policies at time 0 and claim payments occur at time 1. If assets exceed liabilities at time 1, the firm pays the losses and the equity owners receive the residual value (the difference between assets and liabilities). However, if assets are less than liabilities, the insurer defaults and the policyholders receive the assets. The payoff to policyholders at time 1 is thus equal to : L - Max[L-A,0], where L = losses, A = assets,

[3]Sophisticated academic research also has been developed on VaR. One important problem is to allow for time-varying volatility. For example, the volatility of foreign exchange markets may be very high in one period and very low in another period. Another important issue is to allow for auto-regressive patterns in the data, whereby volatility in one period may depend on things which have occurred in prior periods.

and Max[L-A,0] is the payoff on the insolvency put.

The Merton-Perold (M-P) and Myers-Read (M-R) capital allocation techniques take a different view of what is meant by a « marginal » approach to capital.allocation. Stated simply, the M-P approach views marginal capital allocation in terms of what happens to the insolvency put option if entire lines of business are added to or removed from the firm. The M-R approach is marginal in the instantaneous intepretation familiar from calculus, i.e., they allocate capital based on what happens to the firm's insolvency put option in response to very small changes in the liabilities of the lines of business written by the firm. The M-P method may leave some capital unallocated, while the M-R method makes a unique allocation of 100 percent of capital. I now turn to a more detailed examination of each of the concepts and methodologies discussed above.

3.2. Regulatory Risk-Based Capital

My examination of regulatory risk-based capital (RBC) is based on the RBC system used in the United States. RBC has been in effect for life insurers since 1993 and for property-liability insurers since 1994. Even though it is not appropriate to use regulatory capital as a management tool, the regulatory approach is important because it identifies most of the important risks faced by insurers. In addition, regulatory capital may serve as a constraint on certain of the firm's activities, because violation of regulatory solvency boundaries can subject the firm to regulatory costs.

The risk-based capital systems consist of RBC proportions, which are multiplied by various income statement and balance sheet quantities to generate "RBC charges." The sum of the charges equals an insurer's risk-based capital, following a "covariance adjustment" discussed below. The following are the principal risk-based capital charges for property-liability insurers:

Investment RBC = R_1
There exist various charges for bonds, stocks, and other investments. E.g: the charge ratio is zero for Treasury bonds, then progressively higher, capping off at 0.3 for bonds in or near default.

Loss Reserve RBC = R_2

Loss reserve RBC allows for the risk of adverse reserve development, i.e., the possibility that loss payments will exceed reserved amounts. The proportions vary by line that are multiplied by outstanding loss reserves.

Written premium RBC = R_3

Written premium RBC accounts for the possibility that the loss ratio on the new business will be higher than expected. The charges vary by line and are multiplied by net premiums written.

Credit RBC = R_4

Credit RBC allows for the possibility that agents and reinsurers will default on their obligations to pay the company the money they owe.

Off-balance sheet RBC = R_5

Off-balance sheet RBC allows for unexpected payments from contracts that do not show up on the balance sheet such as loan guarantees for subsidiaries. There is no charge for derivatives at the present time, even though derivative transactions may pose a significant off-balance sheet risk for some insurers.

After the risk-based capital charges are determined, they could be added to produce the company's total risk-based capital. However, his would overlook diversification, i.e., it would not recognize the likelihood that an insurer will not be affected by adverse outcomes for all of the risks simultaneously. Indeed, it is possible that an adverse outcome from one type of risk could be offset by a favorable outcome from another. Another way of saying this is that the risks facing insurers are less than perfectly correlated. Because determining the correlations among risks is difficult, the RBC system assumes that the correlations are zero, i.e., that the risks are uncorrelated. This assumption provides the foundation for the RBC formula:

$$R_T = R_0 + [R_1^2+R_2^2+R_3^2+R_4^2+R_5^2]^{1/2}$$

where R_0 = risk-based capital for holdings of stocks of the firm's subsidiaries and R_1 through R_5 are defined earlier. The square root in the formula is a covariance adjustment implicitly assuming zero correlation between risks 1 through 5.

Why would it be a mistake for insurers to use RBC in capital allocation? One reason is that there is no theoretical foundation for the formula. Using a purely empirical approach with no theory would be a mistake. Secondly, the formula is of questionable accuracy (Cummins, Harrington, and Klein 1995). One of the defects of the model is that some of the charges are based on worst-case scenarios, e.g., adverse historical outcomes, rather than utilizing statistical concepts such as variances and covariances to estimate relative risks. The formula also ignores correlations among the firm's businesses, which should be taken into account in an accurate capital allocation system

3.3. The Capital Asset Pricing Model (CAPM)

The CAPM states that the return on equity or cost of capital for a firm is determined by the following formula:

$$r_e = r_f + ß_e [r_m - r_f]$$

where r_e = cost of equity capital,

r_f = default risk-free rate of interest,

r_m = expected return on the "market", and

$ß_e$ = the firm's beta coefficient = $Cov(r_e, r_m)/Var(r_m)$.

where $Cov(\bullet)$ = the covariance operator and $Var(\bullet)$ = the variance operator.

How can the CAPM be used by an insurance company to make pricing and investment decisions? Project decisions can be made by decomposing the beta coefficient to determine the betas by line of business. For example, let's consider an insurer with two lines of business. Its net income would be:

$$I = r_A A + r_1 P_1 + r_2 P_2$$

where I = net income,

r_A = return on assets,

r_1, r_2 = rates of return on underwriting from lines 1 and 2,

A = assets, and

P_1, P_2 = premiums from lines 1 and 2.

Next introduce the balance sheet identity, which says that assets are equal to equity, plus the liabilities generated by the two lines. Then divide by equity to express the result as a rate of return:

$$r_E = r_A(E+L_1+L_2)/E+r_1P_1/E+r_2P_2/E$$

Then the Beta coefficient can be decomposed as follows:

$$ß_E = ß_A(1+k_1+k_2) + ß_1s_1 + ß_2s_2$$

where $ß_E$, $ß_A$, $ß_1$, $ß_2$ = betas for firm, assets, and insurance risk of lines 1 and 2,

k_1, k_2 = liability leverage ratios for lines 1 and 2, = L_i/E, i = 1, 2, and

s_1, s_2 = premium leverage ratios for lines 1 and 2, = P_i/E, i = 1, 2.

The calculation uses the property that the covariance is a linear operator.

The formula for the decomposition of $ß_E$ shows that the beta of the firm, which drives the required return on equity, is the beta of assets times 1, representing the investment of equity capital, plus the liability leverage ratios for lines one and two. Then the formula adds on the beta of each individual line's underwriting returns multiplied by the line-specific premium to surplus ratio. So we find a theoretical justification for the traditional rule of thumb leverage ratio that has been used for years in the insurance industry -- the premium to surplus ratio.

The model can be solved for the required rate of underwriting return on each line of business:

$$r_i = -k_i\, r_f + \beta_i\, (r_m - r_f)$$

for lines of business i = 1 and 2. Thus, each line implicitly pays interest for the use of policyholder funds (the term $-k_i\, r_f$) and receives a rate of return based on the systematic risk of the line (the term $\beta_i\, (r_m - r_f)$).

The CAPM result has the following implication: It is not necessary to allocate capital by line using the CAPM, but rather to charge each line for at least the CAPM cost of capital, reflecting the line's beta coefficient and leverage ratio. Costs of capital based on other asset pricing models, such as the arbitrage pricing theory (APT), have similar implications.

Although the CAPM provides a useful way of conceptualizing the contributions of the firm's lines of business to the return on equity, there are at least three important problems with this model. (1) The CAPM only rewards the firm for bearing systematic underwriting risk, that is, underwriting risk that is correlated with the market portfolio. However, insurers also need to be concerned about extreme events, i.e., tail risk, that is simply not priced in the CAPM model. This is important in view of the role of insurers as financial intermediaries, where a firm's principal creditors are also its customers. (2) Line of business underwriting beta estimates are difficult to estimate given the data currently available (Cummins and Harrington 1988), although progress has been made in estimating costs of capital in the more recent literature (e.g., Lee and Cummins 1998). (3) Research has shown that rates of return are driven by other economic factors besides beta coefficients (e.g., Fama and French 1996). Thus, sole reliance on the CAPM would ignore important determinants of the cost of capital. The primary role for the CAPM is to serve as a benchmark to compare with the results of other estimation methodologies. If the two methods yield drastically different results, it would be appropriate to check the methodology and data carefully before proceeding.

3.4. Value At Risk (VaR)

The V*alue at Risk (VaR)* is defined as the maximum amount the firm could lose over a specified time period with a specified (usually very small) probability. For example, a currency trader might want to know how much could be lost in one week with a probability of 1%. For an insurance company, it might be sufficient to ask how much could be lost in a calendar quarter. For example, at the 1% probability level, the maximum amount a given insurer can lose on a given line of business line 1 over the next six months might be estimated as 5.6 million. The idea is to make sure the insurer's managers understand the degree to which a given line of business is exposing the firm to potentially large losses at any given time.

VaR has become very important in industries such as banking. Although some bank researchers think they have invented something new, in fact concepts similar to value at risk have been used for a long time in the insurance industry. For example, actuaries have been estimating ruin probabilities for many years; and VaR is also similar to the maximum probable loss concept, which is widely used by reinsurers, insurance brokers, and risk managers. The methodologies needed to compute business specific and firm-wide VaR already exist in both the actuarial literature and the financial literature.

How can VaR be used in capital allocation? One approach is to use *exceedence probabilities.* The exceedence probability is defined as the probability that losses from a particular line of business will exceed the expected loss plus the capital allocated to the line. Representing the exceedence probability by ε, it can be defined more formally as follows:

The Exceedence Probability (ε):

$$\text{Probability } [Loss_i > E(Loss_i) + C_i] = \varepsilon$$

where $Loss_i$ = loss from insurance line i,

$E(Loss_i)$ = expected value of loss from line i, and

C_i = capital allocated to line i

Capital can be allocated by equalizing the exceedence probabilities across the lines of business written by the insurer. E.g., for two lines, we would have:

$$\text{Probability } [Loss_1 > E(Loss_1) + C_1] = \varepsilon = \text{Probability } [Loss_2 > E(Loss_2) + C_2]$$

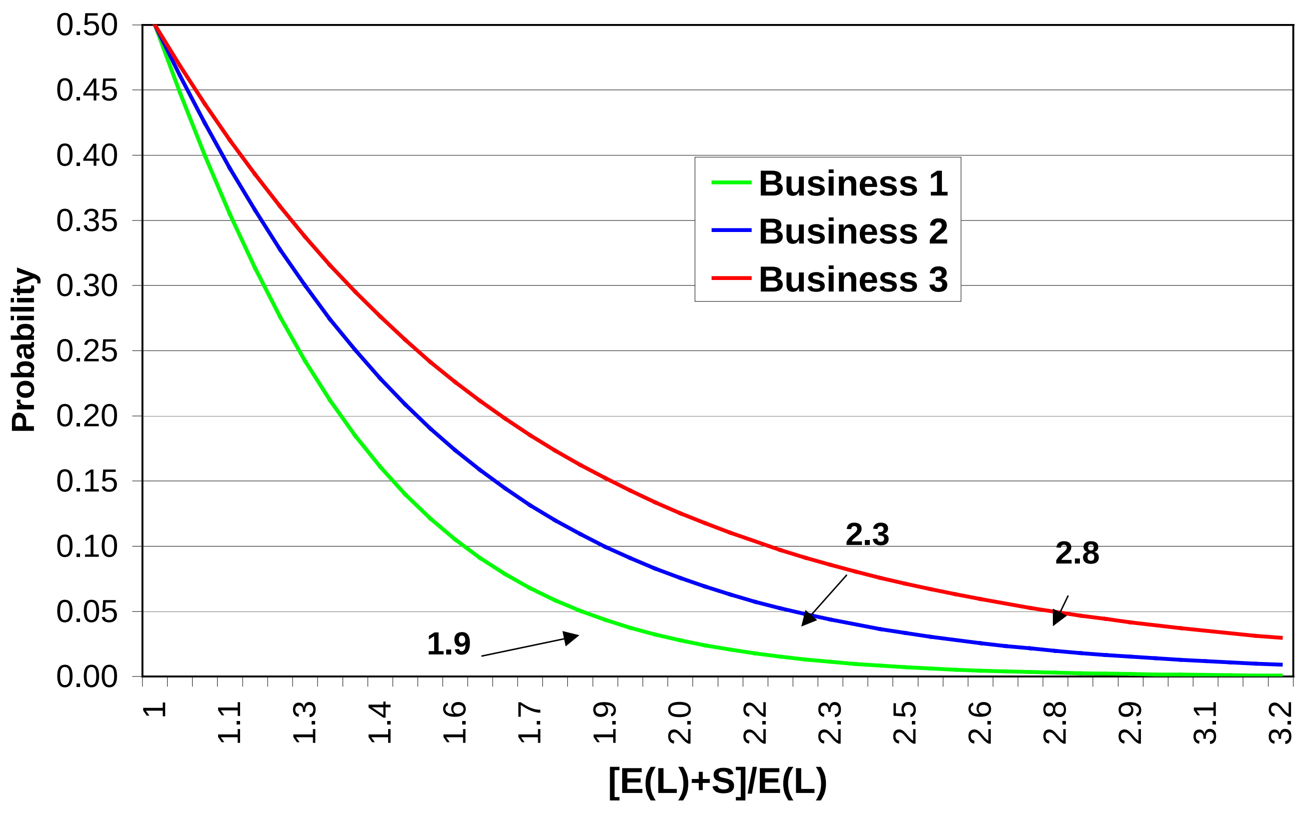
If the lines of business have expected losses that differ in size, it is convenient to express the result in terms of ratios to expected losses:

$$Pr[Loss_1/E(Loss_1) > 1 + C_1/E(Loss_1)] = \varepsilon = Pr[Loss_2/ E(Loss_2) > 1 + C_2/ E(Loss_2)]$$

where the abbreviation Pr is used to represent probability. This formula expresses the required capital in terms of ratios of capital allocations to the expected loss of each line of business. Lines of business with relatively high risk would require more capital relative to expected losses in order to attain the specified exceedence probability. Exceedence probability curves are shown in Figure 1, which plots the amount of loss on the vertical axis against the probability of loss on the horizontal axis.

Three issues to think about in terms of capital allocation using VaR are the following: (1) The firm may not have enough capital to attain a given exceedence probability for all of its businesses. In this case, it can either raise the exceedence probability or raise more capital. (2) The exceedence probability approach as outlined here does not consider the diversification effect across lines. This diversification effect can be taken into account using another method discussed below. (3) The exceedence probability does not tell managers anything about the *amount* by which losses are likely to exceed the available resources if an event occurs that breaches the exceedence level. The latter two issues are addressed by the methodologies discussed below.

Figure 1: VAR Exceedence Probability Curves



3.5. The Insolvency Put Option

Option pricing theory can be used to allocate capital in a manner that is conceptually similar to but more general than value-at-risk analysis. The allocation is conducted using the option model of the firm. Consider an insurance firm with random assets (A) and liabilities (L) that is subject to default risk. The liabilities (policyholder claims) are assumed to be payable one period in the future (i.e., this is a European rather than an American option example). If assets exceed liabilities (A > L) at the liability maturity date, the policyholders receive the value of the liabilities and the owners or residual claimants receive the remaining assets of the firm. However, if the value of the assets is less than the value of the liabilities (A < L), the owners of the firm default on the liability payment and the policyholders receive the assets of the firm.

To allocate capital, we consider the value of the policyholders' claim on the firm at times prior to the maturity of the liabilities. That is, we are seeking to determine the economic value of the policyholders' claim on the firm prior to the liability maturity date. The value of this claim is the risk-less present value of the liabilities, which would be the present value of the liabilities if default risk were zero, minus a "put option," which is a function of assets (A), liabilities (L), the risk-free rate (r), the time to maturity (τ), and volatility (σ). That is, the value of the policyholders' payoff is

$$\text{Value of Policyholders' Claim} = L\, e^{-r\tau} - P(A,L,r,\tau,\sigma)$$

where $P(A,L,r,\tau,\sigma)$ = value of a put option on A with strike price L, interest rate r, time to maturity τ and risk parameter σ. The risk parameter σ reflects the volatility of assets and liabilities, as well as the correlation between them. $P(A,L,r,\tau,\sigma)$ is called the *insolvency put option* or the *expected policyholder deficit (EPD)* (Butsic 1994).

Capital can actually be allocated based on the insolvency put option. This approach is similar to but better than the value-at-risk methodology, because it considers the expected value of the amount that can be lost rather than just giving the probability of exceeding a specified value of loss. In order to illustrate this approach to capital allocation, consider three lines of business with different levels of risk, where the level of risk is determined by the value of the option risk parameter, σ. Here, we will use three lines of business, a low-risk line with σ of 0.375, an intermediate-risk line with σ of 0.5, and a high-risk line with σ of 0.625.[4]

This approach allocates capital to each line such that the insolvency puts or expected policyholder deficits, expressed as a proportion of liabilities, L, are equalized at a specified target level such as 5 percent. To illustrate the method, the put option values for the three businesses are shown in the Figure 2 as a function of the asset-to-liability ratio. The asset-to-liability ratio measures the capital assigned to each line because assets (A_i) are equal to L_i (the nominal loss liability for line i) + C_i (the capital allocated to line i), so the asset-to-liability ratio equals $[1+C_i/L_i]$. As shown in the chart, the value of the EPD or put option for each line declines as the amount of capital allocated to the line increases.

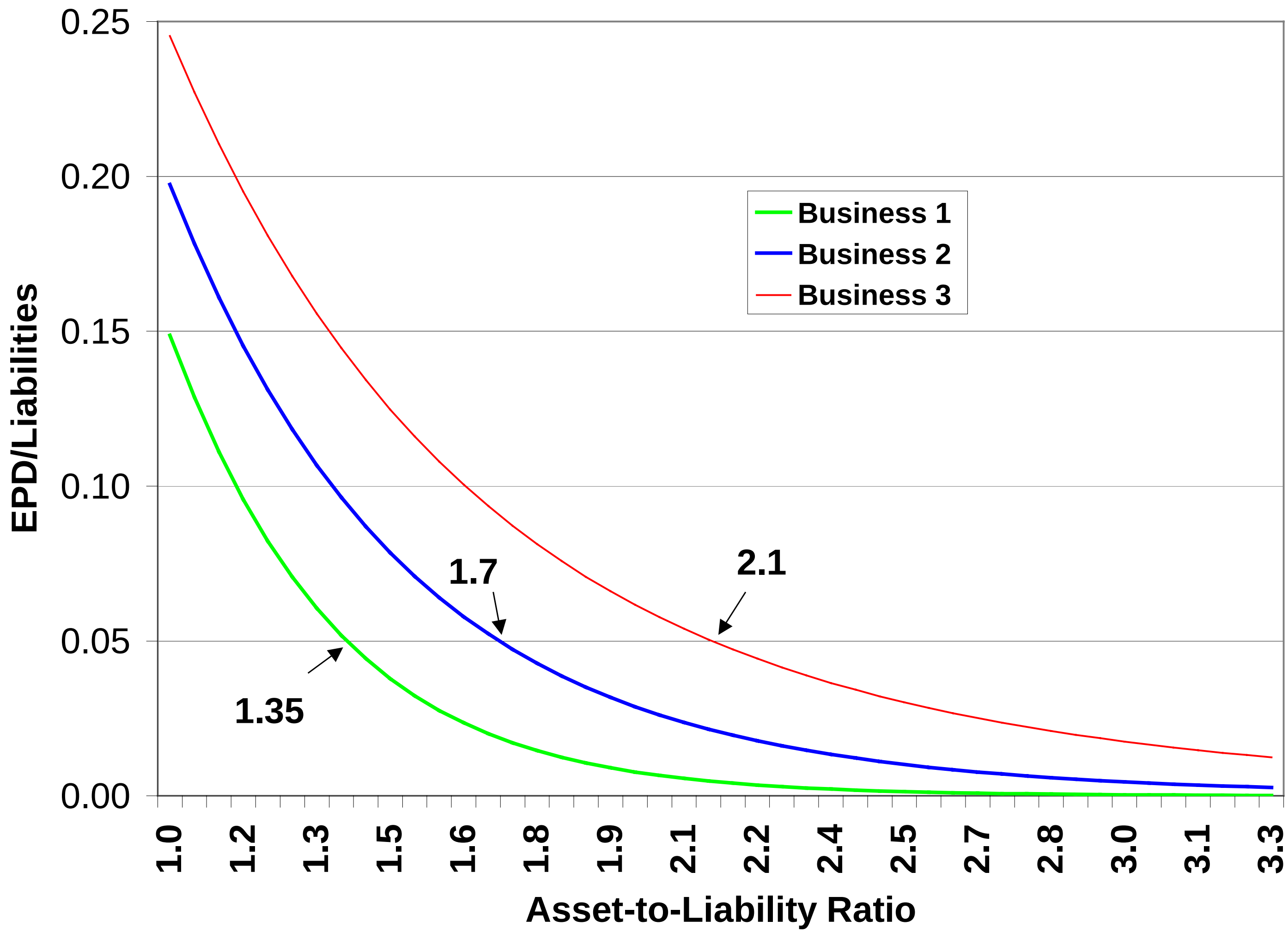
To continue with the capital allocation example, assume that the firm has selected a target EPD of 5% of liabilities. To achieve this target, an asset-to-liability ratio of 1.35 is required for Business 1 (the low-risk line of business). Or, in other words, $0.35 of capital must be allocated to the line for each $1 of nominal loss liability. Similarly, the intermediate-risk line requires an asset-to-liability ratio of 1.7, and the high-risk line requires an asset-to-liability ratio of 2.1. Thus, the amount of capital needed to achieve the EPD objective is directly related to the risk of the line, as measured by the volatility parameter σ.

So, it should be clear that the expected policyholder deficit (EPD), or "insolvency put," allocation method is similar to the value-at-risk (VaR) method. However, the EPD is more informative than VaR because it considers the expected amount of loss that will occur with a specified probability rather than just the amount of loss that will be exceeded with a specified probability. For example, it is more useful to know that the expected loss in excess of some asset-to-liability ratio (e.g., 2.1) will be $5 million rather than just to know that the probability of exceeding the asset-to-liability ratio is 0.05. The firm

[4]In terms of the put option pricing function $P(A,L,r,\tau,\sigma)$, the other parameters used to compute the option values shown in Figure 1 are: $r = 0$, $\tau = 1$, $L = 1$, and A = asset-to-liability ratio. The example is based upon one presented in Merton and Perold (1993). The Black-Scholes option pricing model is used in the example, with $\tau = 1$ and $r_f = 0$. The numbers shown on the graphs have been rounded.

would presumably react differently if the expected amount by which the asset to liability ratio is exceeded is $100 vs. $100 million. (Of course, for some purposes, it would be useful to know both the VaR and the EPD.) The EPD also has the advantage of being consistent with the financial theory of pricing risky debt contracts. This is important because financial theory tells us how to maximize value, or add value to the firm.

Figure 2: Expected Policyholder Deficit



The EPD capital allocation method represents an improvement over value-at-risk. However, a cautionary remark is in order: The method should not be taken to imply that the lines of business are separate from each other in terms of default risk. All lines of business still have access to the firm's entire equity capital, and the firm goes bankrupt overall and not by line. Nevertheless, the method does provide some valuable information as long as the insolvency issue is kept in mind. However, the EPD approach has series defect, i.e., it does not take into account the benefits of diversification across business lines. I next turn to a discussion of the methods that have been suggested to recognize diversification in the option pricing context.

3.6. Recognizing Diversification: Marginal Capital Allocation

It is important to recognize the effects of diversification when allocating capital and measuring returns. An insurer can reduce its overall risk by engaging in a portfolio of businesses whose returns are not perfectly correlated.

Consider first the case where the businesses are all separated into distinct companies, as they would be if a multiple line firm were to spin-off its businesses into separate entities. The spin-off businesses would need more capital to achieve, say, a given VaR or EPD goal, than would the multiple line firm. The multiple line firm would need less capital because of diversification across lines. A particular line may have high stand-alone risk but its return may have a low correlation with returns on other lines of business, making the line very valuable from a diversification perspective. Treating the line on a stand-alone basis would lead to excessive allocations of capital to the line and the underestimation of its risk-adjusted return on capital. Many of the existing capital allocation techniques do not take diversification into account.

How can diversification benefits be recognized when allocating capital? An option-based capital allocation method that recognizes the benefits of diversification has been proposed by Merton and Perold (1993). They base their analysis on the concept of *risk capital*, which is defined as follows:

> Risk Capital is the smallest amount that can be invested to insure the value of the firm's net assets against a loss in value relative to the risk-free investment of those net assets.

The firm's net assets are defined as its gross or total assets minus the default-risk-free value of the firm's policyholder liabilities. Risk capital can be thought of as the value of an option purchased by the firm that guarantees that the firm will receive no less than its net assets at the option maturity date. So if the firm has gross assets (A) of $1,000 and the default-free value of its liabilities is $500, the option pays off if the gross value of assets falls below $1,000. Risk capital can come from various sources. It could be totally supplied by the firm. However, if the firm has default risk, risk capital is partially supplied by the liability holders.

The Merton-Perold methodology can be described by continuing the example used above to illustrate the allocation of capital by expected policyholder deficits. I again consider the three businesses whose put option values are plotted in Figure 2. All businesses are assumed to have nominal liabilities of $1,000, and the volatility parameters used in the illustration are the same as before. However, to consider diversification, assumptions are also needed about the correlations between the lines of business. In the context of the Black-Scholes option model used in the illustration, the relevant correlations are between the natural logarithms of the loss liabilities. I assume the following log correlations: $\rho_{12} = 0.5$, $\rho_{13} = 0.75$, and $\rho_{23} = 0.5$, where ρ_{ij} = log correlation between line i and line j.

To illustrate marginal capital allocation, I first consider the stand-alone capital required for the three business. Stand-alone capital is the capital that would be needed by the firm's businesses if they were operated as separate firms. I assume that all three businesses adopt an expected policyholder deficit target of 5 percent of the nominal (risk-free) value of liabilities. The stand-alone capital for the three firms is shown in Table 1. As Table 1 shows, Business 1 requires assets of $1,361 to attain the 5 percent EPD objective, while Businesses 2 and 3 require assets of $1,672 and $2,107, respectively. These are the same values portrayed at the 5 percent level for the three businesses in Figure 2, although the numbers in the figure have been rounded for presentation purposes.

One could combine the three businesses into a single firm and add the stand-alone risk capital of each of the three businesses to obtain the risk capital of the entire enterprise. However because of

Table 1: Stand-Alone Risk Capital

For EPD = 5% of Liabilities

Business	**Assets**	**Nominal Liabilities**	**Stand-Alone Capital**
1	1,361	1,000	361
2	1,672	1,000	672
3	2,107	1,000	1,107

$\sigma_1 = 0.375$; $\sigma_2 = 0.5$; $\sigma_3 = 0.625$; $\rho_{12} = 0.5$, $\rho_{13} = 0.75$, and $\rho_{23} = 0.5$; $r = 0$; $\tau = 1$.

diversification, the total amount of capital needed in order for the single enterprise to attain the target EPD is less than the sum of the stand-alone capital for the individual lines. In fact, the capital required to achieve the 5% EPD target if the three businesses were combined into a single firm would be $1,427, significantly less than the sum of the stand alone capital of the three businesses ($2,140). The question is how to distribute the benefits of diversification among the three businesses.

The M-P method of capital allocation is conducted in two steps: (1) Calculate the risk-capital required by firms that combine two of the businesses. There are three possible combinations: businesses 1&2, businesses 1&3, and businesses 2&3. (2) Calculate the marginal capital required when the excluded business is added to the two-business firms. I.e., this is the marginal capital required if a firm combining two businesses were to add the third business. The capital allocated to a given business is equal to the marginal capital required when it is added to the appropriate two-business firm. Because the calculation is made for each two firm combination, the method provides a unique capital allocation for each of the three businesses comprising the firm. The order in which the businesses are combined into firms does not matter because all three two-business combinations are used, i.e., the allocated capital of each business is obtained on the assumption that two of the businesses have already been combined.

To illustrate capital allocation, we first consider the firms obtained by forming two-business combinations. The capital required to achieve the 5% EPD target for the two-business firms is shown in Table 2. The table shows the amount of capital needed to achieve the EPD target if businesses 1 and 2 were combined to form a single firm, the amount required if businesses 1 and 3 were combined, and the amount required if businesses 2 and 3 were combined. For each two-business combination, the amount of capital needed for the combined firm is less than the sum of the stand-alone capital of the two businesses comprising the combined firm. This effect occurs because the lines of business are not perfectly correlated.

Table 2: Joint versus Stand-Alone Capital

Example: Effects of Diversification

Business Lines	Joint Capital	Total Stand-alone Capital
1&2	745	1,033
1&3	1,175	1,468
2&3	1,276	1,779

The second step in the M-P capital allocation method is to calculate the additional capital required if the excluded businesses are added to the two-business combination firms shown in Table 2. This step is illustrated in Table 3, which shows the additional capital needed when each business is considered to be the marginal case. The marginal allocations, shown in the column headed "Merton-Perold Marginal Capital," are obtained by subtracting the capital needed for the two-line firm, not including business line i, from the amount of capital needed by the firm consisting of all three lines of business ($1,427). For example, consider the two-business firm consisting of businesses 1 and 2. This firm consisting of lines 1 and 2 needs only $745 in capital to achieve a 5% EPD (see Table 1). Adding business 3 to this firm nearly doubles the required amount of capital. The marginal capital of business 3 is equal to the total capital required for the three-line firm to achieve the 5% EPD target ($1,427) minus the amount of capital needed by the two-line firm consisting of firms 1 and 2 ($745), or $682 = $1,427 - $745. The marginal capital of firms 1 and 2 is computed similarly. The capital allocations to the three lines are directly related to the risk of the lines, with line 3 being given the largest capital allocation because it has the highest risk.

Table 3 also shows that the Merton-Perold marginal capital allocation assigns less than 100 percent of the three-line firm's capital to the three businesses. The total amount allocated is $1,084, which is $343 less than the three-line firm's total required capital of $1,427. It is important to emphasize that the three-line firm requires the entire amount of capital ($1,427) in order to attain the EPD target of 5%. However, the amount of capital allocated to the three businesses marginally is less than the total capital of $1,427. This means that some of the firm's capital is allocated to the «corporate» level rather than to the three individual businesses.

Table 3: Allocated & Unallocated Capital

Current Lines	**Business Added**	**Merton-Perold Marginal Capital**	**Stand-alone Capital**	**Myers-Read Marginal Capital**
1&2	3	682	1,107	811
1&3	2	252	672	392
2&3	1	150	361	224
Totals		1,084	2,140	1,427
Risk Cap for 1,2&3		1,427		
Total Allocated		1,084		
Unallocated		343		

Merton and Perold (1993) argue that capital allocations based on stand-alone capital are likely to lead to incorrect decisions about the projects undertaken by the firm and the performance evaluation of lines of business. As the above example suggests, the M-P method implies that marginal risk capital should be used in the denominator in calculating performance statistics such as the risk adjusted return on capital (RAROC) and economic value added (EVA) for a firm's businesses. Unless the businesses are perfectly correlated (in which case they are actually the same business), this approach results in higher estimates of RAROC and EVA than if the total amount of the firm's capital were allocated to the businesses. M-P argue that a full allocation of capital will lead the firm to reject projects that would add to its market value. Adoption of the marginal approach will lead the firm to make value-maximizing decisions, while use of another approach could lead the firm to avoid some projects that would add to the firm's market value of equity.

Myers and Read (M-R) (1999) also use an option pricing model to allocate capital. However, they reach different conclusions from Merton and Perold. Whereas Merton and Perold allocate capital at the margin by adding entire businesses to the firm (a *macro* marginal allocation), Myers and Read allocate capital by determining the effect of very small changes in loss liabilities for each line of business (a *micro* marginal allocation). Myers and Read show that their method leads to a unique allocation of capital by line that assigns 100 percent of total capital.

The M-R approach can be illustrated by continuing the above example.[5] using the hypothetical firm investigated in this paper. Define the firm's default value (insolvency put option) as $P(L_1,L_2,L_3)$, where $P(\bullet)$ = the insolvency put and L_i = loss liabilities for line i, i = 1, 2, 3, valued at date zero. They then derive capital allocation formulas by taking the derivative of P with respect to each of the L_i . Thus, instead of adding entire businesses to the firm, they perform an incremental analysis using very small changes in L_i. In the case where the objective of the firm is to equalize the marginal default values across lines of business, the formula for the surplus required per dollar of liabilities in line i (s_i) is:

$$s_i = s - (\partial p/\partial s)^{-1}(\partial p/\partial \sigma) \quad [(\sigma_{iL} - \sigma_L^2) - (\sigma_{iV} - \sigma_{LV})]/\sigma$$

where s = the surplus-to-liability ratio of the firm,

σ = firm's overall volatility parameter,

p = the firm's insolvency put per dollar of total liabilities = $P/(L_1+L_2+L_3)$,

$\partial p/\partial s$ = the partial derivative (rate of change) of p with respect to s,

$\partial p/\partial \sigma$ = the rate of change of p with respect to the firm's overall volatility parameter σ,

σ_{iL} = covariance parameter between losses in line i and losses of the entire liability portfolio,

[5] Myers and Read (1999) assume that the firm invests in risky assets and therefore allow for asset volatility as well as covariability between assets and the losses of the three businesses. To keep the example as simple as possible, while focusing on the essential concepts, I have assumed that the firm invests in risk-free assets.

σ_L^2 = volatility parameter for total losses, L,

σ_{iV} = covariance parameter between losses in line i and losses of the asset portfolio, and

σ_{LV} = covariance parameter between the firm's assets and losses.

Thus, because $\partial p/\partial s < 0$ and $\partial p/\partial \sigma > 0$, line i's capital allocation is directly proportional to its covariability with the loss portfolio (σ_{iL}) and inversely proportional to its covariability with the asset portfolio (σ_{iV}). Adding to the covariability of the loss portfolio increases the firm's overall risk level and therefore leads to more capital being allocated to line i. However, because the firm's overall volatility parameter is inversely related to the covariability between assets and liabilities, lines with higher covariability with assets require less equity capital. Intuitively, positive correlation between assets liabilities creates a natural hedge that reduces the risk of the firm.

I calculated the capital allocation for the three-line firm discussed above using the M-R approach. The results are shown in the last column of Table 3, headed "Myers-Read Marginal Capital." It is clear from Table 3 that the M-R method allocates 100% of total capital. Proportionately to the capital allocated by the two methods, the M-R method allocates less to line 3 and more to lines 1 and 2.[6]

Because the amounts of capital allocated to each line of business differ substantially between the M-R and M-P methods, the two methods will not yield the same pricing and project decisions when used with a method that employs by-line capital allocations. Consequently, it is important to determine which method is correct. The M-R method has considerable appeal because it avoids the problem of how to deal with the unallocated capital under the M-P approach. In addition, most decision making regarding pricing and underwriting *is* marginal in the sense of M-R, i.e., typically involving very small changes to an existing portfolio. However, more research is needed to determine which model is more consistent with value maximization.

The conclusions would be the same if asset risk were present in the example.

[6]The calculation is to divide the amount allocated to each line by the total amount allocated: $1,084 for M-P and $1,427 for M-R.

3.7. Additional Issues

Another important issue has to do with the *economic cost* of the firm's overall capital as well as the capital allocated to individual lines of business. The capital of financial institutions such as insurers is invested in marketable securities. If capital markets are efficient and frictionless, the invested funds will earn the equilibrium market rate of return and thus will be cost-less to the firm. However, the existence of various market and institutional imperfections lead to friction costs which imply that the capital invested will not earn the full fair market return required to avoid a loss to the insurer. Various types of friction costs are present that create costs for insurers that reduce the returns from the investment of their capital. The three most important sources of costly capital to insurers are: (1) Agency and informational costs. It is well-known that managers of firms can behave opportunistically, and thus fail to realize the owners' objective of value maximization. In addition, adverse selection and moral hazard are endemic to insurance markets and will create costs to the extent they cannot be controlled through an insurer's pricing and underwriting decisions. (2) The Federal income tax system leads to double taxation of investment income; and, as a result, investing in securities through an insurance company produces lower investment returns than investors could realize by investing directly in the market. And (3) regulation, and especially the risk-based capital system, imposes costs on insurers in the form of a regulatory "option" on the insurer's assets. The option is created because the RBC system gives regulators the legal right to seize control of the insurer when its assets still exceed its liabilities. Other regulations such as investment restrictions also may lead insurers to hold inefficient portfolios, further reducing returns for any given level of risk.

The existence of market frictions means that a spread develops between the returns that could be earned by investing directly in capital markets and the returns actually earned on the capital held by insurers. It is this spread cost that must be taken into account in determining whether lines of business are earning the appropriate rates of return. Of course, the risk of individual lines also is important in determining their cost of capital. Usually, the type of risk recognized in the cost of capital context is systematic market risk, determined by an asset pricing model.[7]

It is also interesting to consider the role of regulatory risk-based capital in the context of a marginal allocation system. With a well-designed marginal capital allocation system, is regulatory risk-based capital relevant? It can be, because there is a potential cost imposed on the firm by the regulatory risk-based capital system. For most insurers, and reasonably small EPD targets, no cost will be realized, since the insurer's total capital generally will be greater than the regulatory capital requirement. We also consider the following two cases: (1) Regulatory capital for one or more lines exceeds the marginal capital allocated to these lines but the insurer's overall capital is greater than risk-based capital. In this case as well regulatory costs are not likely to be incurred because the risk-based capital test is applied to the entire firm and not by line of business. This conclusion would have to be modified if risk-based capital action level tests were to be applied by line. (2) Regulatory risk-based capital exceeds the firm's overall capital, including both allocated and unallocated capital. In this situation, regulatory penalties will apply. Thus, the conclusion is that regulatory capital usually will not be a problem, even if one or more lines of business have allocated capital that is less than the by-line risk-based capital, as long as the firm's overall capital exceeds its overall risk-based capital.

The final point to be made is a caveat. Insurers should use caution in designing and implementing a capital allocation system. The use of an inappropriate system is likely to lead to rejection of some projects that should be accepted and acceptance of some projects that should be rejected. Systems that use stand-alone capital are likely to be particularly harmful. However, the adoption of an appropriate marginal capital allocation system can have significantly beneficial effects on the market value of the firm, by enabling the firm to identify projects and businesses that are creating value for shareholders as well as those that are destroying firm value.

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[7] For further discussion of cost of capital estimation in insurance, see Cummins and Harrington (1987) Cummins and Lamm-Tennant (1994) and Lee and Cummins (1998).

5. Conclusion

It is possible to draw some conclusions about the methodologies that have been discussed in this article. The first is that the expected policyholder deficit provides more information than value at risk. Consequently, the focus should be on the EPD although it may also be useful to calculate VaR. Additionally, exceedence probability and EPD graphs convey much more information than looking at a single VaR or EPD value.

Secondly, using an option model that recognizes diversification will lead to better decisions than VaR or EPD-based methodologies because diversification is an important benefit from operating a multiple line firm. Between the two approaches that recognize diversification, the Myers-Read method seems to make the most sense because most pricing and underwriting decisions are marginal in the sense considered in their methodology. However, more research is clearly needed to determine whether the Myers-Read model actually is more consistent with value-maximization than competing models. A third conclusion is that the cost of capital allocated to a line is the «spread cost,» i.e., the cost over and above the cost of capital if this capital were not held in an insurance company but invested directly in the capital market.

Fourth, capital allocation must consider both asset and liability risk and allow for covariability between assets and liabilities. Capital allocations are increasing in the covariability between a line's losses and the company's overall losses and decreasing in the covariability between a line's losses and the company's asset portfolio. Fifth, the allocation of capital should consider the duration and maturity of liabilities. Even if allocation is done in terms of the liability line lending money to the asset department, the liability line still needs to be charged based upon its duration. It is necessary to work out so-called transfer pricing schemes. In this regard, it may be useful to study the public utility literature where transfer-pricing models are discussed.

Sixth, the decision-making system should drive the design of the data system, and not the other way around. Inevitably, when one starts out, the data will drive the system, but when data systems are re-designed one needs to think about what kind of information is needed to use capital allocation models. Annual data are insufficient to obtain accurate capital allocations. Therefore, it is critical to obtain data as frequently a possible. Finally, the winning firms in the Twenty-First Century will be the ones that successfully implement capital allocation and other financial decision-making techniques. Such firms will make better pricing, underwriting, and entry/exit decisions and create value for shareholders.

References

Albrecht, Peter, «Risk Based Capital Allocation and Risk Adjusted Performance Management in Property/Liability Insurance: A Risk Theoretical Framework,» Proceedings of the 75h International AFIR Colloquium (1997, Cairns, Australia).

Butsic**,** Robert, 1994, « Solvency Measurement for Property-Liability Risk-Based Capital Applications, » *Journal of Risk & Insurance* 61 : 656-690.

Cummins, J. David and Scott E. Harrington, 1988 «The Relationship Between Risk and Return: Evidence from Property-Liability Insurance Stocks,» *Journal of Risk and Insurance* 55: 15-31.

Cummins, J. David, Scott E. Harrington, and Robert W. Klein, 1995, "Insolvency Experience, Risk-Based Capital, and Prompt Corrective Action in Property-Liability Insurance," *Journal of Banking and Finance* 19: 511-527.

Cummins, J. David, Scott E. Harrington, and Greg Niehaus, 1993, "An Economic Overview of Risk-based Capital Requirements in the Property-liability Insurance Industry, *Journal of Insurance Regulation* 11: 427-447.

Cummins, J. David and Joan Lamm-Tennant, 1994, "Capital Structure and the Cost of Equity Capital in Property-Liability Insurance." *Insurance: Mathematics and Economics* 15 (December 1994): 187-201.

Fama**,** Eugene F. and Kenneth R.French, 1996, « The CAPM Is Wanted, Dead or Alive, » J*ournal of Finance* 51 : 1947-1958.

Kaplan, Paul D and James D. Peterson, 1998, « Full-Information Industry Beta**, »**.*Financial Management* 27 : 85-93. Summer 1998.

Alice C. Lee and J. David Cummins, 1998, "Alternative Models for Estimating the Cost of Equity Capital for Property/Casualty Insurers,"*Review of Quantitative Finance and Accounting* 10: 235-267.

Matten, Chris, Managing Bank Capital: Capital Allocation and Performance Measurement (New York: John Wiley, 1996).

Merton, Robert and Andre Perold, «Theory of Risk Capital In Financial Firms,» Journal of Applied Corporate Finance (1993).

Myers, Stewart C. and James A. Read, Jr., 1999, "Surplus Allocation for Insurance Companies," working paper, Massachusetts Institute of Technology, Cambridge, MA.

Zaik, Edward; John Walter; Gabriela Kelling; and Christopher James, «RAROC At Bank of America: From Theory to Practice,» Journal of Applied Corporate Finance (1996).