Comparative Mixed Risk Aversion: Definition and Application to Self-Protection and Willingness to Pay

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Abstract

Mixed risk aversion (Caballé and Pomansky, 1996) defines the class of increasing utility functions that have derivatives alternating in sign, with positive odd derivatives and negative even derivatives. In this article, we characterize comparative mixed risk aversion so as to answer the following question: how different risk attitudes affect choices when expenditures change event probabilities? Attempts to answer this question in the literature found an endogenous probability. We show that the threshold probability is ½ under mixed risk aversion. We consider applications to self-protection and willingness to pay. We obtain that if agent *v* is more mixed risk averse than agent *u*, then *v* will select a higher level of self-protection and will have a higher willingness to pay than *u* only if the accident probability is lower than ½. We extend the results to the presence of a background risk.

Keywords: Mixed risk aversion, comparative mixed risk aversion, self-protection, willingness to pay, background risk.

JEL classification: D80.

Résumé

L'aversion au risque mélangée (Caballé et Pomansky, 1996) définit la classe des fonctions d'utilité croissantes qui ont des dérivées alternant en signe, avec des dérivées impaires positives et des dérivées paires négatives. Dans cet article, nous caractérisons l'aversion au risque mélangée comparée (entre différents individus) afin de répondre à la question suivante : comment des attitudes différentes face au risque influencent les choix de dépenses qui, elles, affectent les probabilités d'accident ? Des études récentes visant à répondre à cette question ont trouvé une probabilité endogène comme probabilité frontière permettant de séparer les prédictions. Nous montrons que la probabilité frontière est ½ sans l'hypothèse d'aversion au risque mélangée. Nous appliquons notre modèle à l'autoprotection et à la volonté à payer. Nous montrons que si l'agent *v* est plus riscophobe que l'agent *u*, alors *v* choisira un niveau d'autoprotection plus élevé et aura une plus grande volonté à payer pour réduire la probabilité d'accident que l'agent *u*, seulement si la probabilité d'accident est inférieure à ½. Nous étendons nos résultats à des situations risquées avec un risque latent.

Mots clés : Aversion au risque mélangée, aversion au risque mélangée comparée, autoprotection, volonté à payer, risque latent.

Classification JEL : D80.

1 Introduction

For many economic applications under risk and uncertainty, a simple concave transformation of a von Newmann-Morgenstern utility function (or an Arrow-Pratt increase in risk aversion) does not yield intuitive changes in the decision variables that affect event probabilities or distribution functions.

For example, following the contribution of Ehrlich and Becker [1972] who introduced the concepts of self-protection and self-insurance into the literature, Dionne and Eeckhoudt [1985] showed that a more risk averse individual, in the sense of Arrow-Pratt, does not necessarily produce more self-protection activities than a less risk averse one[1]. In other words, he does not modify his optimal action in an intuitive direction.

A second example concerns the willingness to pay literature (Drèze, 1962; Jones-Lee, 1974; and Pratt and Zeckhauser, 1996). One can easily verify that a more risk averse decision maker in the Arrow-Pratt sense is not necessarily willing to pay more for a lower probability of death or for a lower probability of accident than a less risk averse individual (Eeckhoudt, Godfroid and Gollier, 1997). In a third example, McGuire, Pratt and Zeckhauser [1991] showed that more risk averse individuals may choose more risky decisions (described as less insurance and more gamble) than less risk averse individuals. They obtained that this behavior depends upon a critical switching probability.

In the previous three examples, the individual decisions imply first order shifts instead of pure second order ones such as a mean preserving spread (Rothschild and Stiglitz, 1970). Moreover, as we shall see, their actions usually affect higher moments of the random variable distribution when appropriate restrictions are not imposed[2]. Consequently, to make predictions on (risk averse) decision makers' behaviors, one needs restrictions either on utility or on distribution functions that take into account actions, such as self-protection, that may affect all distribution moments. In this paper, we shall concentrate on restrictions related to utility functions. For an analysis of restrictions on distribution functions see Julien, Salanié and Salanié [1999], and for restrictions on the loss functions see Lee [1998].

In Section 2 we introduce the concept of "More Mixed Risk Aversion" and we present a transformation theorem that sets a sufficient condition to compare mixed risk averse utility functions. By definition, individual v is more mixed risk averse than individual u if he is more risk averse, more prudent, more temperate… or if the absolute ratio of the n^{th+1} derivative over the n^{th} of individual v is higher than the corresponding ratio of individual

[1] On this issue see also Briys and Schlesinger [1990], Julien, Salanié and Salanié [1999], Chiu [2000], Gollier and Eeckhoudt [2001], and Lee [1998]. In fact, one cannot make any prediction on how a more risk averse agent will choose his optimal level of effort in a principal-agent relationship without introducing strong assumptions on the utility function (Arnott, 1992).

[2] For the self-protection example, the i^{th} moment of the gross expected loss is $p(x)l^i$, where p is the probability of accident, x is the level of self-protection and l is the amount of loss in case of accident. For the principal-agent problem where the outcomes distribution can be written as $F(l/x)$ it is also clear that the first derivative $F_x(l/x)$ does not affect only the mean of l.

u for all *n* greater than one. As applications, we shall show how the concept of more mixed risk aversion is useful to establish an exogenous threshold probability over which a more mixed risk averse agent invests less in self-protection activities and has a lower willingness to pay.

In Section 3 we show that if agent *v* is more mixed risk averse than agent *u*, then *v* selects a higher level of self-protection or has a higher willingness to pay only if the accident probability is lower than ½[3]. This result is important since the great majority of risky situations that require self-protection (occupational safety, firearm safety, road safety, health care, environmental prevention, …) and public decisions on safety are characterized for events with a probability lower than ½. We also obtain that the switching probability of becoming a gambler is greater than ½ in the probability-improving environment of McGuire, Pratt and Zeckhauser[4]. In Section 4 we extend the above results to risky situations with a background risk (Doherty and Schlesinger, 1983; Eeckhoudt and Kimball, 1992). Concluding remarks are presented in Section 5.

2 Mixed risk aversion

2.1 Definition

Caballé and Pomansky [1996] generalized the Arrow-Pratt index of absolute risk aversion to higher order. They defined the n^{th} order index of absolute risk aversion as

$$A_n^u(w) = -\frac{u^{(n+2)}(w)}{u^{(n+1)}(w)}, \text{ for } n \geq 0,$$

and defined mixed risk aversion as:

Definition 1 *(Caballé and Pomansky,* 1996*) A real-valued continuous utility function u defined on* $[0,\infty)$ *exhibits mixed risk aversion if and only if it has a completely monotone first derivative on* $(0,\infty)$ $\left(i.e.\ (-1)^n\ u^{(n+1)}(w) \geq 0,\ for\ \ n \geq 0\right)$ *and u* $(0) = 0$.

Theorem 2.2 in Caballé and Pomansky [1996] states that *u(w)* is a mixed risk aversion function if and only if it admits the following representation

$$u(w) = \int_0^\infty \frac{1-e^{-wt}}{t}\, dF_u(t), \qquad (1)$$

with

[3] Julien, Salanié and Salanié [1999] derived simultaneously and independently a similar result. However, they did not study the existence of an exogenous bound that will be effective for all risk averse agents and for all levels of loss.

[4] In their model, activity *x* increases the winning probability instead of decreasing the probability of loss as in the self-protection and willingness to pay applications.

$$\int_1^{\infty} \frac{dF_u(t)}{t} < \infty .$$

In this case the n^{th} order index of absolute risk aversion, $A_n^u(w)$, can be written as:

$$A_n^u \ (w) = \frac{\int_0^{\infty} t^{n+1} \ e^{-wt} \ dF_u(t)}{\int_0^{\infty} t^n \ e^{-wt} \ dF_u(t)}.$$

As pointed out by Pratt and Zeckhauser [1987], most of the utility functions commonly used in economics and finance such as the logarithmic and the power functions are in the class of completely monotone functions. These functions have the property of being characterized by the measure describing the mixture of exponential functions.

Mixed risk aversion can be characterized equivalently as

i) $A_n^u(\cdot)$ is decreasing in *w* for all *w* and *n.*
or
ii) $A_n^u(w) \leq A_{n+1}^u(w)$ for all *w* and *n.*

The equivalence between *i*) and *ii)* follows from $A_n^u(w) \geq 0$ and the identity

$$\frac{d}{dw} A_n^u(w) = A_n^u(w)\left(A_n^u(w) - A_{n+1}^u(w)\right).$$

To prove that $A_n^u(\cdot)$ is decreasing in *w* we apply Cauchy-Schwartz inequality, i.e.,

$$\left(\int \varphi \ (t) \ \psi \ (t) \ dF_u \ (t)\right)^2 \leq \int \varphi^2 \ (t) \ dF_u \ (t) \int \psi^2 \ (t) \ dF_u \ (t),$$

to

$$\varphi(t) = t^{(n+2)/2} \ e^{-wt/2}, \ \psi(t) = t^{n/2} \ e^{-wt/2}.$$

2.2 Mixed risk aversion and other concepts of attitude toward risk

In 1987, Pratt and Zeckhauser introduced the concept of Proper Risk Aversion in order to make predictions on lottery choices in the presence on an independent, undesirable lottery. According to their definition, a utility function is proper when an undesirable lottery can never be made desirable by the presence of another (independent) undesirable risk. Their concept is preserved in the class of utility functions for which derivatives alternate in sign, with positive odd derivatives and negative even derivatives. They showed that mixtures of exponential utilities are proper and that properness implies decreasing absolute risk aversion (DARA)[5]. Brockett and Golden [1987] developed a parallel characterization of such functions and Hammond [1974] proposed a first application using a mixture (discrete) of exponential functions.

Caballé and Pomansky [1996] have shown that mixed risk aversion implies standard risk aversion (Kimball, 1993) which in turn implies properness (Pratt and Zeckhauser, 1987). As discussed by Gollier and Pratt [1996], properness implies risk vulnerability. However, standardness, properness and risk vulnerability were mainly developed to take into account different forms of background risks and are not directly useful for the purpose of comparing self-protection activities and willingness to pay decisions' among risk averse individuals.

The reader may verify that the functional representation in (1) is equivalent to the two representations of mixture of exponential utilities proposed by Pratt and Zeckhauser [1987, equations (12) and (13)]. These equations in Pratt and Zeckhauser [1987] are sufficient for Proper Risk Aversion while (1) is necessary and sufficient for mixed risk aversion.

2.3 Comparative mixed risk aversion

Let us consider two risk averse agents *u* and *v*. Following Pratt [1964], it has been established that comparative risk aversion amounts to applying a simple concave transformation *k* of a utility function: *v* is more risk averse than *u* if and only if $v = k(u)$ with $k'' < 0$. However, this type of comparison is not sufficient for problems that imply the variation of all moments of the distribution such as self-protection or willingness to pay. We introduce the next definition.

Definition 2 *Let u and v be two mixed risk averse utility functions. We say that v is more mixed risk averse than u if and only if* $A_n^u(w) \leq A_n^v(w)$*, for all n and w.*

We can show the following result that will be useful for the comparison of optimal decisions between different mixed risk averse individuals.

Theorem *Let u and v be two mixed risk averse utility functions described respectively by distribution functions* F_u *and* F_v*. If* $dF_v(\cdot)$ *dominates* $dF_u(\cdot)$ *in the sense of the Maximum Likelihood Ratio (i.e.* $\frac{dF_u(\cdot)}{dF_v(\cdot)}$ *is decreasing over* $(0,\infty)$*) then v is more mixed risk averse than u.*

Before presenting the proof, let us consider an example that will provide the intuition of the theorem.

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[5] See Gollier and Pratt [1996] for a comparison of three concepts proposed in the recent literature and related to the willingness to accept a risk when another independent background risk is added to random wealth: risk vulnerability, properness and standardness. In this article, we consider only one random variable. See however Section 4.

Consider

$$u\ \ (w) = -p_1 e^{-a_1 w} - p_2 e^{-a_2 w} - \ldots - p_n e^{-a_n w}$$
$$v\ \ (w) = -q_1 e^{-a_1 w} - q_2 e^{-a_2 w} - \ldots - q_n e^{-a_n w},$$

with p_i, q_i, and a_i as positive parameters for i = 1, …, n and $a_1 < a_2 < \ldots < a_n$. If $\frac{p_1}{q_1} \geq \ldots \geq \frac{p_n}{q_n}$, then from the theorem we know that v is more risk averse, more prudent, more temperate than u, and more generally, more mixed risk averse, that is

$$A_n^u(\cdot) \leq A_n^v(\cdot), \text{ for all } n \geq 0 .$$

The intuition behind the theorem is quite simple. Observe that u and v are mixtures of CARA utility functions. Consider for the sake of illustration the case where there are only two positive a_i (a_1 and a_2) in the example above. By transforming p_1 into q_1 lower than p_1, less weight is put upon the less risk averse CARA component of the u function (since $a_1 < a_2$). Of course lowering p_1 also implies that q_2 exceeds p_2 so that simultaneously more weight is placed upon the more risk averse component of u. As result v is surely more risk averse than u and the Theorem shows that this property automatically extends to all ratios of the successive derivatives of each utility function.

Since mixed risk averse utility functions are in the class of DARA, we can make use of a result from Jewitt [1987] to prove the theorem by extending his proof to all $n \geq 0$.

Proof :

To prove the theorem we need to show that $(-1)^n\ v^{(n)}$ is more risk averse than $(-1)^n\ u^{(n)}$, or that $\int_0^\infty I_n(w)dF_v(t)$ is more risk averse than $\int_0^\infty I_n(w)dF_u(t)$ for all $n \geq 0$, where $I_0(w) = \frac{1-e^{-wt}}{t}$ and $I_n(w) = -t^{n-1}e^{-wt}$ for $n \geq 0$.

Since, $I_n(w)$ corresponds to a constant risk averse utility function for all $n \geq 0$, if $dF_v(\cdot)$ dominates $dF_u(\cdot)$ in the sense of the Maximum Likelihood Ratio, then it follows, from Jewitt [1987], that $\int_0^\infty I_n(w)dF_v(t)$ is more mixed risk averse than $\int_0^\infty I_n(w)dF_u(t)$.

This completes the proof of the theorem.

As already pointed out[6] the result of the theorem is a special case of Jewitt [1987] which states that, under DARA, a Maximum Likelihood Ratio change in background risk raises the aversion to other independent risks. In Section 4, we shall show that the theorem can be extended to the presence of a background risk.

3 Applications

We now apply the comparative mixed risk aversion result first to decisions by mixed risk averse agents on self-protection and then to willingness to pay. Intuitively, we expect a more mixed risk averse individual to exert more self-protection activities and to be willing to pay a higher monetary amount for a lower probability of accident.

3.1 Self-protection

The standard model for self-protection (Ehrlich and Becker, 1972) can be summarized as follows. Consider an individual with an increasing von-Neuman-Morgenstern utility function u and a non-random initial wealth w_0. The agent faces a risk of total loss l and can invest a quantity x in self-protection activities, in order to reduce the probability of loss $(p(x))$, a decreasing function of x. The action cost c for one unit of x is fixed at $c \equiv 1$. With two states of the world the optimal choice of self-protection will be a solution of:

$$\max_x \; p(x)u(w_0 - l - x) + (1 - p(x))u(w_0 - x), \tag{2}$$

subject to the constraint $x \geq 0$. The first-order condition for an optimal choice x_u^* requires

$$0 = p'(x)\left[u(w_0 - l - x) - u(w_0 - x)\right] - \left[p(x)u'(w_0 - l - x) + (1 - p(x))u'(w_0 - x)\right].$$

The second order necessary condition is

$$\begin{aligned} p''(x)\left[u(w_0 - l - x) - u(w_0 - x)\right] - 2p'(x)\left[u'(w_0 - l - x) - u'(w_0 - x)\right] \\ + p(x)u''(w_0 - l - x) + (1 - p(x))u''(w_0 - x) \leq 0. \end{aligned}$$

Note that risk aversion is not sufficient to yield the second order condition negative (see Arnott, 1992, for details). In the remainder of this article we assume that all conditions for allowing the solution of (2) to be a global maximum are met. Consequently all the derived results are restricted to these conditions as it is usual in this literature.

Self-protection activities do not necessarily reduce risk, but do affect the probabilities of the various states as well as their contingent outcomes. The problem here is different from that where probabilities are fixed as in the context of market insurance. One

[6] We thank C. Gollier for making this observation.

consequence is that more risk averse agents in the sense of Arrow-Pratt will not necessarily choose a higher level of self-protection spending (see Dionne and Eeckhoudt, 1985, for explicit examples). McGuire, Pratt and Zeckhauser [1991] found an endogenous critical switching probability that depends on preferences and outcomes, and interpret expenditures as gambling or insurance. This endogenous switching probability is retrieved by Lee [1998]. In this section, we show that the endogenous probability is lower than ½ for self-protection and willingness to pay and greater than ½ in the probability improving environment of McGuire, Pratt and Zeckhauser [1991].

Julien, Salanié and Salanié [1999] showed that if v is more risk averse than u in the sense of Arrow-Pratt, then there exists a threshold probability $\overline{p}$ such that self-protection is higher for v than for u if and only if the probability of loss resulting from the optimal choice of u is less than $\overline{p}$, with

$$\overline{p} = \frac{\sum_{i=1}^{\infty} \frac{(-1)^i}{i!} l^i \left[v' u^{(i)} - u' v^{(i)} \right]}{\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} (-1)^i \, (-1)^j \, \frac{l^i l^j}{i! \, j!} \left[v^{(i)} u^{(j+1)} - u^{(i)} v^{(j+1)} \right]}. \tag{3}$$

The problem with this switching probability $(\overline{p})$ is that it is endogenous since it depends on u, v and on outcomes. The critical value $\overline{p}$ is then different on a case by case basis, which makes the result of a little value. We can show the next proposition for the class of mixed risk averse utility functions.

Proposition 1 *Suppose the probability of loss p(x) is decreasing in x and let u and v be two mixed risk averse utility functions. Let* x_u^* *and* x_v^* *be the optimal level of effort selected respectively by agents u and v. Let v be more mixed risk averse than u, and suppose* x_v^* *is higher than* x_u^**, then* $p(x_u^*) \leq 1/2$.

Proposition 1 can be stated equivalently as: if $p(x_u^*) \geq 1/2$, then $x_u^* \geq x_v^*$. That is prevention can decrease with mixed risk aversion if the probability of loss is sufficiently high. For example, when $p(x_u^*) \geq 1/2$, an increase in risk aversion and in prudence, both induced by an increase in mixed risk aversion, reduces prevention.

Proof of Proposition 1:

Proving Proposition 1 is equivalent to prove that $\overline{p} \leq ½$.

Let's denote

$$K_{ij} = (-1)^i \, (-1)^j \left[v^{(i)} u^{(j+1)} - u^{(i)} v^{(j+1)} \right].$$

It can be easily shown that:

$$K_{ij}\begin{cases} >0 \; if \; i<j+1 \\ =0 \; if \; i=j+1 \\ <0 \; if \; i>j+1 \end{cases}. \tag{4}$$

The denominator in (3) can be written as:

$$-l\sum_{i=1}^{\infty}\frac{(-1)^i}{i!}\,l^i\left[v'u^{(i+1)}-u'v^{(i+1)}\right]+\sum_{i=2}^{\infty}\sum_{j=1}^{\infty}\frac{l^{i+j}}{i!\,j!}K_{ij} \;\;,$$

with $-l\sum_{i=1}^{\infty}\frac{(-1)^i}{i!}\;l^i\;\left[v'\;\;u^{(i+1)}-u'\;\;v^{(i+1)}\right]\geq 0$ from (4).

Now we prove that

$$\sum_{i=2}^{\infty}\sum_{j=1}^{\infty}\frac{l^{i+j}}{i!\,j!}K_{ij}\;\;\geq 0\,. \tag{5}$$

First, note that

$$\begin{aligned} K_{ij} &= (-1)^i\;(-1)^j\;\left[\;v^{(i)}\;\;u^{(j+1)}-u^{(i)}\;\;v^{(j+1)}\right] \\ &= -(-1)^i\;(-1)^j\;\left[\;u^{(i)}\;\;v^{(j+1)}-v^{(i)}\;\;u^{(j+1)}\right] \\ &= -(-1)^{i-1}\;(-1)^{j+1}\;\left[\;u^{(i)}\;\;v^{(j+1)}-v^{(i)}\;\;u^{(j+1)}\right] \\ &= -K_{j+1,\,i-1}, \end{aligned} \tag{6}$$

and hence

$$\frac{1}{i!\,j!}K_{ij}+\frac{1}{(j+1)\;!\;(i-1)\;!}K_{j+1,i-1}=(\frac{1}{i!\,j!}-\frac{1}{(j+1)\;!\;(i-1)\;!})K_{ij},$$

Moreover, since $i > j + 1$ if and only if $\frac{1}{i!\;j!}<\frac{1}{(j+1)!(i-1)!}$, and from (4) we have

$$\frac{1}{i!\,j!}K_{ij}+\frac{1}{(j+1)\;!\;(i-1)\;!}K_{j+1,i-1}\geq 0. \tag{7}$$

Next we prove that (7) is sufficient to get (5).

We can write $\sum_{i=2}^{n}\sum_{j=1}^{n}\frac{l^{i+j}}{i!\,j!}K_{ij}$ as

$$\sum_{i=2}^{n}\sum_{j=1}^{n}\frac{l^{i+j}}{i!\,j!}K_{ij}=\sum_{k=3}^{2n}l^{k}\Big(\sum_{\substack{i+j=k,\\ i\geq 2,\, j\geq 1}}\frac{1}{i!\,j!}K_{ij}\Big). \tag{8}$$

Depending on whether k is odd or even, the term inside the summation in (8) can be written as:

- k is even $(k=2p,\ p\geq 2)$.

$$\begin{aligned}\sum_{\substack{i+j=2p,\\ i\geq 2,\, j\geq 1}}\frac{1}{i!\,j!}K_{ij}&=\sum_{\substack{i+j=2p,\\ i\leq p, i\geq 2,\, j\geq 1}}\frac{1}{i!\,j!}K_{ij}+\sum_{\substack{k+l=2p,\\ k\geq p+1, k\geq 2, l\geq 1}}\frac{1}{k!\,l!}K_{kl}\\ &=\sum_{\substack{i+j=2p,\\ i\leq p, i\geq 2,\, j\geq 1}}\frac{1}{i!\,j!}K_{ij}+\sum_{\substack{k+l=2p,\\ l\leq p-1, k\geq 2, l\geq 1}}\frac{1}{k!\,l!}K_{kl}.\end{aligned} \tag{9}$$

Redefining indexes in the second term of the right hand in the previous equation as $i=l+1$ and $j=k-1$, (9) can be written as

$$\sum_{\substack{i+j=2p,\\ i\leq p, i\geq 2,\, j\geq 1}}\frac{1}{i!\,j!}K_{ij}+\sum_{\substack{i+j=2p,\\ i\leq p, i\geq 2, i\geq 1}}\frac{1}{(j+1)!(i-1)!}K_{j+1,i-1}=\sum_{\substack{i+j=2p,\\ i\leq p, i\geq 2, i\geq 1}}\Big(\frac{1}{i!\,j!}K_{ij}+\frac{1}{(j+1)!(i-1)!}K_{j+1,i-1}\Big),$$

which is positive from (7).

- k is odd $(k=2p+1,\ p\geq 1)$.

Since $K_{p+1,p}=0,$ it follows that

$$\begin{aligned}\sum_{\substack{i+j=2p+1,\\ i\geq 2,\, j\geq 1}}\frac{1}{i!\,j!}K_{ij}&=\sum_{\substack{i+j=2p+1,\\ i\leq p, i\geq 2,\, j\geq 1}}\frac{1}{i!\,j!}K_{ij}+\sum_{\substack{k+l=2p+1,\\ k\geq p+2, k\geq 2, l\geq 1}}\frac{1}{k!\,l!}K_{kl}\\ &=\sum_{\substack{i+j=2p+1,\\ i\leq p, i\geq 2,\, j\geq 1}}\frac{1}{i!\,j!}K_{ij}+\sum_{\substack{k+l=2p+1,\\ l\leq p-1, k\geq 2, l\geq 1}}\frac{1}{k!\,l!}K_{kl}.\end{aligned}$$

Once again redefining indexes in the second term in the right hand of the previous equation as $i=l+1$ and $j=k-1$, we get

$$\sum_{\substack{i+j=2p+1,\\ i\le p, i\ge 2, j\ge 1}} \frac{1}{i!\,j!}K_{ij} + \sum_{\substack{k+l=2p+1,\\ l\le p-1, k\ge 2, l\ge 1}} \frac{1}{k!\,l!}K_{kl} = \sum_{\substack{i+j=2p,\\ i\le p, i\ge 2, i\ge 1}} \left(\frac{1}{i!\,j!}K_{ij} + \frac{1}{(j+1)!(i-1)!}K_{j+1,i-1}\right),$$ which is also positive from (7).

As a result

$$\forall \;\; n \ge 2,\; \sum_{i=2}^{n}\sum_{j=1}^{n} \frac{l^{i+j}}{i!\,j!}\;K_{ij} \ge 0,$$

and at the limit we obtain

$$\sum_{i=2}^{\infty}\sum_{j=1}^{\infty} \frac{l^{i+j}}{i!\,j!} K_{ij} \;\; \ge 0\,.$$

The denominator in (3) is the sum of two positive terms. We can then write

$$\overline{p} \le \frac{\sum_{i=1}^{\infty} \frac{(-1)^i}{i!} l^i \left[v' u^{(i)} - u' v^{(i)}\right]}{\sum_{i=1}^{\infty} \frac{(-1)^{i+1}}{i!} l^{i+1} \left[v' u^{(i+1)} - u' v^{(i+1)}\right]}$$

or

$$\overline{p} \le \frac{\sum_{i=2}^{\infty} \frac{(-1)^i}{i!} l^i \left[v' u^{(i)} - u' v^{(i)}\right]}{\sum_{i=2}^{\infty} \frac{(-1)^i}{(i-1)!} l^i \left[v' u^{(i)} - u' v^{(i)}\right]}.$$

Since for $i \ge 2$, $i! > 2(i-1)!$, and since $(-1)^i\, l^i \left[v' u^{(i)} - u' v^{(i)}\right] > 0$, we then have

$$\frac{(-1)^i}{i!} l^i \left[v' u^{(i)} - u' v^{(i)}\right] < \frac{1}{2}\frac{(-1)^i}{(i-1)!} l^i \left[v' u^{(i)} - u' v^{(i)}\right].$$

Taking the summation over $i \ge 2$ gives $\overline{p} \le$ ½.

By symmetry it can be obtained that $\overline{p} \ge$ ½ when the winning probability $p(x)$ is increasing in x as in McGuire, Pratt and Zeckhauser [1991]. The mathematical development is identical to that made in Proposition 1 with $1 - p(x)$ being the probability of loss for the modified problem. We have the next result:

Corollary *Under the same notation as in Proposition 1, with an increasing winning probability,* $p(x)$*, if* $x_u^* \le x_v^*$, then $p(x_u^*) \ge 1/2$.

3.2 Willingness to pay

Willingness to pay (*WTP*) is a guideline for public and private investment policies and according to *WTP*, public investment projects, such as health care, environmental prevention or road safety investments will be recommended only if the total of monetary amounts rendered to the different agents benefiting from favorable probability changes exceeds the capital cost of the project concerned. Alternative resource allocations are also compared on the basis of *WTP*.

In other situations it is more appropriate to offer different bundles of risk to different individuals if valuations of risk differ among agents. To establish such bundles, one then needs to know the *WTP* for the different risk averse categories. It is this last point that we shall discuss in the remainder of this section. Suppose that risk-reducing benefits are privately valued (medical expenses). As pointed out by Pratt and Zeckhauser [1996], individuals with 'high valuation of risk reduction' would choose expensive plans which may lead to services that are ineffective to most individuals. As we did for the self-protection model we shall use the concept of more mixed risk aversion to order *WTP* values. We can show the next result.

Proposition 2 *Let u and v be two mixed risk averse utility functions and* WTP_u*,* WTP_v *their corresponding amounts of willingness to pay. Let v be more mixed risk averse than u, then* WTP_u is smaller than WTP_v *only if the probability of loss corresponding to the willingness to pay choice of u is lower than ½.*

Proof of Proposition 2:

The expected utility for *u* is:

$$U = pu(w_0 - l) + (1-p)u(w_0)$$

and for individual *v*

$$V = pv(w_0 - l) + (1-p)v(w_0).$$

In order to obtain the willingness to pay for *u* (Drèze, 1962; Jones-Lee, 1974), we completely differentiate *U* with respect to *p* and w_0 to obtain:

$$WTP_u = \frac{dw_0}{dp} = \frac{u(w_0) - u(w_0 - l)}{pu'(w_0 - l) + (1-p)u'(w_0)} \tag{10}$$

The same result holds for individual *v*

$$WTP_v = \frac{dw_0}{dp} = \frac{v(w_0) - v(w_0 - l)}{pv'(w_0 - l) + (1-p)v'(w_0)} \tag{11}$$

The threshold probability $\overline{p}$ is solution of (10) = (11):

$$\overline{p} = \frac{v'(w_0)\,\Delta u - u'(w_0)\Delta v}{\Delta u'\,\Delta v - \Delta v'\,\Delta u} \tag{12}$$

With the Taylor expansion we have

$$\Delta u = \sum_{i=1}^{\infty}(-1)^i \frac{l^i}{i!} u^{(i)}(w_0 - x),\ \Delta v = \sum_{i=1}^{\infty}(-1)^i \frac{l^i}{i!} v^{(i)}(w_0 - x).$$

We can then rewrite (12) as:

$$\overline{p} = \frac{\sum_{i=1}^{\infty} \frac{(-1)^i}{i!} l^i \left[v' u^{(i)} - u' v^{(i)}\right]}{\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}(-1)^i(-1)^j \frac{l^i l^j}{i!\,j!}\left[v^{(i)}u^{(j+1)} - u^{(i)}v^{(j+1)}\right]}$$

The remainder of the proof to obtain that $\overline{p} \leq$ ½ is the same as in Proposition 1.

4 Background risk

In this section we consider the case where the individual faces a background risk $(\tilde{\varepsilon})$ on wealth that is independent of the occurrence of an accident. Let's denote $\tilde{u}(w) = E_\varepsilon(u(w+\varepsilon)) = \int u(w+\varepsilon)\,dF(\varepsilon)$. We know that an individual with a utility function *u* and a background risk $\tilde{\varepsilon}$ behaves as an individual with utility function $\tilde{u}$ and no background risk. Kimball [1993] showed that if *u* has a decreasing absolute risk aversion and a decreasing absolute prudence, then these properties hold for $\tilde{u}$. In other words if *u* is standard risk averse then $\tilde{u}$ is also standard risk averse. We know that mixed risk aversion implies standardness. Consequently, if we suppose that *u* is mixed risk averse, then $\tilde{u}$ is also mixed risk averse and hence, for all $n \geq 0$, $-\frac{\int u^{(n+2)}(w+\varepsilon)\,dF(\varepsilon)}{\int u^{(n+1)}(w+\varepsilon)\,dF(\varepsilon)}$ is decreasing in *w* (following *i)* in page 5), which is an extension of Proposition 4 in Kimball [1993] to mixed risk aversion. Consequently, we can state the following proposition.

Proposition 3 *Let u and v be two mixed risk averse functions and suppose that v is more mixed risk averse than u, then* $\tilde{u}$ *and* $\tilde{v}$ *are mixed risk averse functions and* $\tilde{v}$ *is more mixed risk averse than* $\tilde{u}$.

A detailed proof of Proposition 3 is in Dachraoui et al. [1999].

Proposition 3 allows us to extend the results of Section 3 directly to situations with a background risk. For example if the probability of loss resulting from the optimal self-protection choice of agent u is higher than ½, and if agent v is more mixed risk averse than u, then even in the presence of a background risk, it follows from Propositions 1 and 3 that agent v selects a smaller self-protection effort than agent u. By analogy, Proposition 2 can also be extended to a background risk.

5 Conclusion

In this article we have characterized comparative mixed risk aversion. We have shown how this comparison of attitudes to risk can be useful in ordering optimal decision variables that affect all distribution moments among different mixed risk averse individuals. We showed that more mixed risk averse individuals select higher effort and have a higher willingness to pay only if the probability of accident is lower than ½.

Many extensions of this article can be considered. First it would be interesting to analyze how our measure of comparative attitude to risk generated by more mixed risk aversion can be useful in predicting the agent's action in a principal-agent framework when utility functions are not additively separable. How do different mixed risk averse agents choose the optimal level of effort when faced with a given risk sharing contract? A more difficult question would be to compare how different risk sharing contracts are handled by different mixed risk averse agents.

Another extension is related to the willingness to pay literature. Up to now, since it was not possible to know in what circumstance one could compare the willingness to pay amounts of different risk averse individuals, it was also not possible to aggregate these amounts. Such aggregation is now possible for the class of mixed risk averse utility functions since we have established an exogenous probability that does not depend on preferences nor on wealth.

References

Arnott, R., 1992, "Moral Hazard and Competitive Insurance Markets," in Dionne G. (ed.) Contributions to Insurance Economics, Boston: Kluwer Academic Press, 325-358.

Briys, E. and Schlesinger, H., 1990, "Risk-Aversion and the Propensities for Self-Insurance and Self-Protection," Southern Economic Journal, 57, 458-467.

Brockett, P. L. and Golden, L. L., 1987, "A Class of Utility Functions Containing All the Common Utility Functions," Management Science, 33, 955-964.

Caballé, J. and Pomansky, A., 1996, "Mixed Risk Aversion," Journal of Economic Theory, 71, 485-513.

Chiu, W.H., 2000, "On the Propensity to Self-Protect," Journal of Risk and Insurance, 67, 555-577.

Dachraoui, K., Dionne, G., Eeckhoudt, L. and Godfroid, P., 1999 "Proper Risk Behavior," Working paper 99-01, Risk Management Chair, HEC Montréal (http://www.hec.ca/gestiondesrisques/99-01.pdf).

Dionne, G. and Eeckhoudt, L., 1985, "Self-Insurance, Self-Protection and Increased Risk-Aversion", Economics Letters, 17, 39-42.

Doherty, N. and Schlesinger, H., 1983, "Optimal Insurance in Incomplete Markets", Journal of Political Economy, 91, 1045-1054.

Drèze, J., 1962, "L'utilité sociale d'une vie humaine,"Revue française de recherche operationelle 6, 93-118.

Eeckhoudt, L, Godfroid, P. and Gollier, C., 1996, "Willingness to Pay, The Risk Premium and Risk Aversion," Economics Letters, 55, 355-360.

Eeckhoudt, L. and Gollier, C., 2001, "The Impact of Prudence on Optimal Prevention," Mimeo, Université de Toulouse, France.

Eeckhoudt, L. and Kimball, M., 1992, "Background Risk, Prudence and the Demand for Insurance," in G. Dionne (ed.), Contributions to Insurance Economics, Boston: Kluwer Academic Press, 239-254.

Ehrlich, I. and Becker, G., 1972, "Market Insurance, Self-Insurance and Self-Protection," Journal of Political Economy 80, 623-648.

Gollier, C., and Pratt, J.W., 1996, "Risk Vulnerability and the Tempering Effect of Background Risk," Econometrica 64, 1109-1123.

Hammond, J. S., 1974, "Simplifying the Choice Between Uncertain Prospects when Preference is Non-Linear," Management Science, 20, 1047-1072.

Jewitt, I., 1987, "Risk Aversion and the Choice Between Risky Prospects: The Preservation of Comparative Statics Results," The Review of Economic Studies, 54, 73-85.

Jones-Lee, M., 1974, "The Value of Changes in the Probability of Death or Injury," Journal of Political Economy, 82, 835-849.

Julien, B., Salanié, B., and Salanié, F., 1999, "Should More Risk-Averse Agents Exert More Effort?," Geneva Papers on Risk and Insurance Theory, 24, 19-28.

Kimball, M. S., 1993 "Standard Risk Aversion," Econometrica 61, 589-611.

Lee, K., 1998, "Risk Aversion and Self-cum-Protection," Journal of Risk and Uncertainty, 17, 139-150.

McGuire, M., Pratt, J. and Zeckhauser, R., 1991, "Paying to Improve Your Chances: Gambling or Insurance?," Journal of Risk and Uncertainty, 4, 329-338.

Pratt, J. W., 1964, "Aversion to One Risk In the Presence of Others," Journal of Risk and Uncertainty, 1, 395-413.

Pratt, J. W. and Zeckhauser, R., 1987, "Proper Risk Aversion," Econometrica, 55,143-154.

Pratt, J. W. and Zeckhauser, R., 1996, "Willingness to Pay and the Distribution of Risk and Wealth," Journal of Political Economy, 104, 747-763.

Rothschild, M. and Stiglitz, J., 1970, "Increasing Risk: A Definition," Journal of Economic Theory 2, 225-243.